## Fourth Test

Tuesday, April 19, 2005

You are allowed to use a TI-30Xa/TI-36X (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use one side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.
do NOT Write on this question paper, Which must be turned in at the end of the test (but NOT stapled to your solutions)

1. Given that $f(1)=5, f^{\prime}(1)=8$ and $f^{\prime \prime}(t)=2+\frac{3}{\sqrt{t}}$ for all $t>0$, find $f(t)$ exactly [12]
2. Use the substitution $u=\sqrt{2 x-3}$ to find the exact value of $I=\int_{2}^{6} \frac{3 x-2}{\sqrt{2 x-3}} d x$
3. Let $R$ be the region bounded to the left by the $y$-axis, above by the line $y=x+2$ and below by the parabola $y=x^{2}$. Find the exact area of $R$

4. Let $R$ be the region bounded to the left by the $y$-axis, above by the line $y=x+2$ and below by the parabola $y=x^{2}$. If $R$ is rotated about the $y$-axis, find the exact volume of the solid thus generated
(a) by integrating with respect to $x$ and
(b) by integrating with respect to $y$

Hint: You may assume that the volume of a cone of height $h$ and base radius $r$ is $\frac{1}{3} \pi r^{2} h$.
5. Use L'Hôpital's rule to calculate $\lim _{x \rightarrow 0} f(x)$ where $f(x)=\frac{1}{\ln (1+x)}-\frac{1}{x}$
[Perfect score: $2 \times 12+8+16+12=60]$

