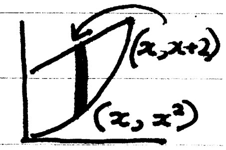
1. $f''(t) = 2 + 3t^{-1/2} \Rightarrow f'(t) = 2t + 6t^{1/2} + b \Rightarrow f(t) = t^2 + 4t^{3/2} + bt + c$ where b and c are arbitrary constants.
But $f(1) = 5$, $f'(1) = 8 \Rightarrow 1 + 4 + b + c = 5$, $2 + 6 + b = 8 \Rightarrow b = 0$, $c = 0$. Hence $f(t) = t^2 + 4t\sqrt{t}$.

2. $u = \sqrt{2x-3} \Rightarrow u^2 = 2x - 3 \Rightarrow x = \frac{1}{2}u^2 + \frac{3}{2} \Rightarrow \frac{dx}{du} = u$

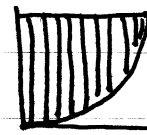
So $$I = \int_{u=\sqrt{2\cdot 2-3}}^{u=\sqrt{2\cdot 6-3}} \frac{3x-2}{\sqrt{2x-3}} \frac{dx}{du}\, du = \int_{u=1}^{u=3} \frac{3\{\frac{1}{2}u^2 + \frac{3}{2}\} - 2}{u}\, u\, du$$
$$= \int_1^3 \left(\frac{3}{2}u^2 + \frac{5}{2}\right) du = \left(\frac{1}{2}u^3 + \frac{5u}{2}\right)\Big|_1^3 = \frac{3^3}{2} + \frac{5\cdot 3}{2} - \frac{1}{2} - \frac{5}{2} = 18$$

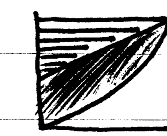
3. $\delta A = (x + 2 - x^2)\delta x + o(\delta x) \Rightarrow A = \int_0^2 (x + 2 - x^2)\, dx$
$= \left(\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3\right)\Big|_0^2 = 2 + 4 - \frac{8}{3} - 0 = \frac{10}{3}$

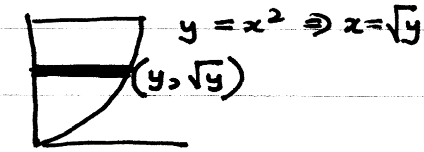


4. (a) Now (with same generic strip) we have $\delta V = 2\pi$ radius height $\delta t + o(\delta t)$
$= 2\pi x\{x + 2 - x^2\}\delta x + o(\delta x) \Rightarrow V = \int_0^2 2\pi x(x + 2 - x^2)\, dx =$
$2\pi \int_0^2 (x^2 + 2x - x^3)\, dx = 2\pi\left\{\frac{1}{3}x^3 + x^2 - \frac{1}{4}x^4\right\}\Big|_0^2 = 2\pi\left\{\frac{8}{3} + 4 - 4 - 0\right\} = \frac{16\pi}{3}$

(b) Let $||||$, $\equiv$ and $\blacksquare$ generate V_2, V_1 and V, respectively. Then $\equiv$ is a cone with $r = 2$ and $h = 2$, implying $V_1 = \frac{1}{3}\pi 2^2 \cdot 2 = \frac{8\pi}{3}$.







For $||||$ we have $\delta V_2 = \pi\, \text{radius}^2\, \delta t + o(\delta t)$
$= \pi x^2 \delta y + o(\delta y) \Rightarrow V_2 = \int_{y=0}^{y=4} \pi x^2\, dy$
$= \pi \int_0^4 x^2\, dy = \pi \int_0^4 (\sqrt{y})^2\, dy = \pi \int_0^4 y\, dy = \pi\left(\frac{1}{2}y^2\right)\Big|_0^4 = \pi\left\{\frac{1}{2}4^2 - 0\right\}$
$= 8\pi$. So $V = V_2 - V_1 = 8\pi - \frac{8\pi}{3} = \frac{16\pi}{3}$ as before.

5. $f(x) = \frac{g(x)}{h(x)}$ where $g(x) = x - \ln(1+x)$ and $h(x) = x\ln(1+x) \Rightarrow g(0) = 0 - \ln(1) = 0 = 0\ln(1) = h(0)$

We have $g'(x) = 1 - \frac{1}{1+x} \Rightarrow g'(0) = 1 - 1 = 0$ and $h'(x) = 1\cdot\ln(1+x) + x\cdot\frac{1}{1+x}$
$\Rightarrow h'(0) = \ln(1) + 0 = 0$. Also, $g'(x) = -(1+x)^{-1} \Rightarrow g''(x) = (1+x)^{-2}$
$\Rightarrow g''(0) = (1+0)^{-2} = 1$ and $h'(x) = \ln(1+x) + \frac{x}{1+x} \Rightarrow$
$h''(x) = \frac{1}{1+x} + \frac{1}{(1+x)^2} \Rightarrow h''(0) = 1 + 1 = 2$. So, by L'Hôpital's rule, $g(0) = 0 = h(0)$ AND $g'(0) = 0 = h'(0)$ AND $h''(0) \neq 0 \Rightarrow$
$$\lim_{x\to 0} f(x) = \frac{g''(0)}{h''(0)} = \frac{1}{2}.$$