MAC 2311, Sections 09 & 10, Spring 2005 (Dr M-G)

Final examination

Friday, April 29, 2005

You are allowed to use a TI-30Xa/TI-36X (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part thereof) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that

$$\frac{d}{du}\{\arctan(u)\} = \frac{1}{1+u^2}$$

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. Calculate: **(a)** $\int_0^4 \left|x^2 - 4\right| dx$ [5] **(b)** $F'(t)$ for $F(t) = \int_1^{\sqrt{t}} \sin(x^2)\, dx$ [4]

2. If $f''(t) = \frac{1}{t^2}$ for all $t > 0$, $f'(1) = 2$ and $f(1) = 3$, find $f(t)$ exactly [9]

3. What is the tangent line to the curve $y = (2x+5)e^{3x}$ at the point $(0, 5)$? [9]

4. A decelerating particle moves away from the origin with displacement $x = \frac{\ln(1+2t)}{1+2t}$ at time t. What is the exact time at which its velocity is zero? [9]

5. Use the substitution $u = \sqrt{x-1}$ to find the exact value of $I = \int_2^4 \frac{1}{x\sqrt{x-1}}\, dx$ [9]

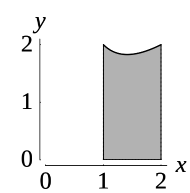
6. A function f is defined on $[1, 4]$ by $f(x) = \begin{cases} 11 + \ln\left(\frac{27}{x^3}\right) - \frac{6}{x} & \text{if } 1 \le x < 3 \\ 3x & \text{if } 3 \le x \le 4 \end{cases}$

(a) Verify that f is continuous [3]
(b) Find and identify all local extrema [8]
(c) Find both global extrema of f on $[1, 4]$, together with their extremizers [4]

Note that $\ln(27) < 4$

7. R is the region enclosed by $y = 0$, $y = x + \frac{2}{x} - 1$, $x = 1$ and $x = 2$. Find:

(a) The area of R [3]
(b) The volume generated by rotating R about the y-axis [6]
(c) The volume generated by rotating R about the x-axis [6]



8. Use L'Hôpital's rule to calculate $\lim_{x\to 0} \frac{\ln(1+x) - \arctan(x)}{\cos(2x) - e^x + \sin(x)}$ [15]

[Perfect score: $5 \times 9 + 3 \times 15 = 90$]