1. (b) Put \( w = \cos(x) \) so that \( \frac{d}{dw} (w) = \frac{d}{dw} \{\cos(x)\} \)

\[
\Rightarrow 1 = -\sin(x) \frac{dx}{dw} .
\]

Then

\[
\int_{x=0}^{x=\pi/4} \frac{\sin(x)}{\cos(x)} \, dx = \int_{w=\cos(0)}^{w=\cos(\pi/4)} \frac{\sin(x)}{\cos(x)} \, dw
\]

\[
= \int_{w=1}^{w=\sqrt{2}} \left\{ -\sin(x) \frac{dx}{dw} \right\} \, dw = -\int_{1}^{\sqrt{2}} \frac{1}{w} \, dw
\]

\[
= -\left[ \ln(w) \right]_{1}^{\sqrt{2}} = \ln(1) - \ln\left(\sqrt{2}\right)
\]

\[
= 0 + \ln\left(\sqrt{2}\right) = \ln(2^{1/2})
\]

\[
= \frac{1}{2} \ln(2)
\]

because \( \ln(a^b) = b \ln(a) \) and because of this

3. Put \( w = \pi x \) so that \( x = w/\pi \Rightarrow \frac{dx}{dw} = \frac{1}{\pi} \)

Then

\[
\int_{x=0}^{x=\pi/2} \cos(\pi x) \, dx = \int_{w=0}^{w=\pi/2} \cos(\pi x) \frac{dx}{dw} \, dw
\]

\[
= \int_{0}^{\pi/2} \cos(w) \frac{1}{\pi} \, dw = \frac{1}{\pi} \int_{0}^{\pi/2} \cos(w) \, dw = \frac{1}{\pi} \sin(w) \bigg|_{0}^{\pi/2}
\]

\[
= \frac{1}{\pi} \left\{ \sin\left(\frac{\pi}{2}\right) - \sin(0) \right\} = \frac{1}{\pi} \left\{ 1 - 0 \right\} = \frac{1}{\pi}.
\]