Second Assignment

Due at 1:25 p.m. on Monday, March 6, 2017

1. Use the corner conditions to carefully *deduce*[¶] a unique admissible broken extremal for the problem of minimizing

$$J[y] \;=\; \int_0^2 y^2(1-y')^2\,dx$$

subject to $y(0)=0$ and $y(2)=1$. [10]

2. For the problem of minimizing

$$J[y] \;=\; \int_0^b \frac{1+y^2}{y'^2}\,dx$$

subject to $y(0)=0$ and $y(b)=\sinh(b)$, find all $b>0$ such that an admissible extremal satisfies both Legendre's and Jacobi's necessary condition. [10]

3. For the problem of minimizing

$$J[x] \;=\; \int_0^2 \sqrt{1+x^2\dot{x}^2}\,dt$$

subject to $x(0)=1$ and $x(2)=3$:

(a) Find the unique admissible extremal.

(b) Show that it satisfies Weierstrass's necessary condition directly, that is, use (10.24)-(10.25), not (10.27). [10]

4. For the problem of minimizing

$$J[x] \;=\; \int_0^2 \sqrt{1+\left(\frac{\dot{x}}{x}\right)^2}\,dt$$

subject to $x(0)=1$ and $x(2)=3$:

(a) Find the unique admissible extremal.

(b) Show that it satisfies Weierstrass's necessary condition directly, that is, use (10.24)-(10.25), not (10.27). [10]

5. For the problem of minimizing

$$J[y] \;=\; \int_a^b \{y'^4 - y'^2\}\,dx$$

subject to $y(a)=\alpha$ and $y(b)=\beta$,

(a) Find a condition that determines when a simple broken extremal exists.

(b) Verify that the condition holds when $a=0=\alpha$, $b=2$ and $\beta=1$. Find all simple broken extremals for this particular case. [10]

[Perfect score: $5\times 10=50$]

[¶]No credit for simply knowing the answer already from Assignment 1.