1 Introduction and Overview

War has been defined as "organised violence carried on by political units against each other" (Bull, 1977, p. 184) or—far less succinctly, but more explicitly—as

> a contest of long or short duration involving armed clashes in one or more theatres between organized units which, in turn, rest upon the support (logistical, manpower) of a home base or bases, and which may be in theatre or outside, with the same home bases and their lines of linkage to forces in theatre also liable to armed attack (Bellany, 1999, p. 729).

Whatever war is, you know it when you see it, and it "has been with us ever since the dawn of civilization. Nothing has been more constant in history than war" (Aumann, 2006). So, in a sense, all efforts at peacemaking since the dawn of time have failed. But why—surely not for want of trying? Aumann[1] has a thesis about that. He argues, in effect, that despite much laudable effort at resolving specific conflicts, far too little effort has been devoted to

> studying war as a general phenomenon, studying its general, defining characteristics, what the common denominators are, what the differences are. Historically, sociologically, psychologically, and—yes—*rationally* ...we should start studying war, from all viewpoints, for its own sake. Try to understand what makes it happen. Pure, basic science. *That* may lead, eventually, to peace (Aumann, 2006, p. 17075).

What motivates this course is a desire to contribute to such an understanding, accompanied by a firm belief that mathematical models have a central role to play in this endeavor. Nevertheless, it is well to appreciate that mathematical models are also only part of understanding that process, and must typically be supplemented by judicious verbal analysis. As suggested by Shubik (1983, pp. v-vi),

> ...the formal modeling of conflict has become both more sophisticated and promising. A blend of mathematics, insight, verbal description and perception of operational relevance is evolving. Essays and numbers; history and equations must be melded to produce insights which could not be obtained without the intermix of quantification and qualification.

That was more than three decades ago. The promise for formal modelling of conflict may since have greatly increased—indeed, if it hadn't, I would scarcely be offering this course!—but the intermix is just as important now as it was then, and likely always will be. Put differently, mathematical modelling is a fourfold process of (i) abstraction from empirical knowledge, (ii) formulation, (iii) analysis and (iv) interpretation in the light of empirical knowledge—and the process requires skill in all four domains.

Yet if anything can be said to be the essence of mathematical modelling, then it is surely abstraction, which means taking away—as Maynard Smith (1972, p. 21) has said, "all good models in science leave out a lot. A model which included everything would be too complicated to analyze." Indeed no model is too simple if it yields a useful answer to a relevant question. To illustrate, let us follow Wright (1965, p. 1272) and Cioffi-Revilla (1989, p. 565) by considering the overall probability P of war during an extended period of time containing N independent crises, each of which may escalate to war with probability $p \in (0,1)$. Elementary probability theory yields

$$P \ = \ 1 - (1-p)^N \tag{1.1}$$

[1] Robert J. Aumann, who shared the 2005 Nobel Prize for Economics with Thomas C. Schelling.

from which two things are clear at once. First, this is a very simple expression. Second, it increases rapidly towards 1 either as p increases for fixed N or as N increases for fixed p: it is not good news for either p or N to increase. But which would be worse—more crises, or a higher escalation probability for each? Put differently, what has a greater impact in terms of keeping the peace—a reduction of p, or a reduction of N? The answer is not immediately obvious. (Is it?)

It helps to answer this question—and many another question—if we first restate it. Let us instead ask: what infinitesimal proportionate increase of the dependent variable P is induced by an infinitesimal proportionate increase of an independent variable, either p or N? The limiting ratio between these two quantities is known as the elasticity of the dependent variable with respect to the independent one, and it measures the sensitivity of the first to a change in the second. We can therefore ask instead: which is higher, the elasticity of P with respect to p, which we denote by e_p, or the elasticity of P with respect to N, which we denote by e_N? The first of these two elasticities is the limiting ratio of

$$\frac{\frac{\Delta P}{P}}{\frac{\Delta p}{p}} \;=\; \frac{p}{P}\frac{\Delta P}{\Delta p}$$

Because P varies smoothly with respect to p (for any N), we obtain

$$e_p \;=\; \lim_{\Delta p \to 0} \frac{p}{P}\frac{\Delta P}{\Delta p} \;=\; \frac{p}{P}\frac{\partial P}{\partial p} \;=\; \frac{Np(1-p)^{N-1}}{P} \tag{1.2}$$

Likewise, the second elasticity is the limiting ratio of

$$\frac{\frac{\Delta P}{P}}{\frac{\Delta N}{N}} \;=\; \frac{N}{P}\frac{\Delta P}{\Delta N}$$

Because P varies discretely with respect to N (for any p), however, the smallest possible ΔN is $\Delta N = 1$, corresponding to $\Delta P = \{1-(1-p)^{N+1}\} - \{1-(1-p)^N\}$ for any value of N. Hence the limiting ratio is

$$e_N \;=\; \frac{N}{P}\frac{\Delta P}{\Delta N}\bigg|_{\Delta N=1} \;=\; \frac{N}{P}\frac{\{1-(1-p)^{N+1}\} - \{1-(1-p)^N\}}{1} \;=\; \frac{Np(1-p)^N}{P} \tag{1.3}$$

Comparing (1.3) to (1.2), we find that invariably $e_p > e_N$. holds. So P is always more sensitive to p than it is to N: efforts to reduce the probability of escalation during crises will always have a greater impact on reducing the overall probability of war than efforts to reduce the frequency of crises. As Cioffi-Revilla notes,

> This is a nice result, since . . . N seems uncontrollable . . . whereas the influence of mediators and peacemakers, the pressure to prevent war (exerted by public opinion, other nations or groups), the availability of information technology aimed at decreasing uncertainty and fear (e.g. data channels for rapid communication), are but a few of the possible ways, or policy instruments, which can be used to lower p (Cioffi-Revilla, 1989, p. 567).

But it becomes a nice result only when supplemented by judicious interpretation. The importance of this point cannot be overemphasized: clever interpretation is just as big a part of clever modelling as clever formulation or analysis.

A is for **Assume**. What you are given is rarely enough, so you will have to make assumptions—about what is important and what is not, about what is assured beyond reasonable doubt and what is still open to question. Indeed, in a very real sense, a model is simply the assumptions you make. Mathematics enables you to deduce, from those assumptions, conclusions which (a) might otherwise not be so readily apparent and (b) can be compared with observations of the real phenomenon that your abstract model attempts to explain. The degree of correspondence determines the value of the model. Poor agreement does not (or should not!) suggest that the mathematics is wrong, however, but rather that one (or more) of the assumptions you made is of doubtful validity. Then you modify your model (i.e., modify your assumptions), and the merry-go-round begins again.

B is for **Borrow**. Why borrow? A mathematical model is an attempt to capture, in abstract form, the essential characteristics of an observed phenomenon. The success of the attempt depends as much (if not more) on the modeller's empirical knowledge of that phenomenon as on her or his mathematical ability. What do you do if you have neither the knowledge nor the time to acquire it? The answer is that you borrow your assumptions, from the scientific literature or from more experienced colleagues (being sure, of course, to acknowledge your sources). With due respect to geniuses, it is much more practical to build upon existing models than it is to start from scratch. So you must be prepared to borrow freely. Yet therein lies a danger, the danger that you will accept too readily the authority of the printed word. Hence ...

C is for **Criticize**. You must be prepared to criticize, too—prepared to criticize not only your own assumptions but also those you have borrowed from other people. Who is to say if they are right or wrong? The answer is you! Modelling is an iterative process. You begin by assuming or borrowing; with the help of mathematics, you reach conclusions; you criticize them; if you are not satisfied then you assume or borrow again, conclude and criticize again, and so on, until eventually you are satisfied (that the model explains the observations). Don't forget the ABC. Assume. Borrow. Criticize.

Table 1.1: An ABC of Modelling. Adapted from Mesterton-Gibbons (2007, p. xix)

Our modus operandi for the course is based on Mesterton-Gibbons (2007), in which the practice of modelling is viewed as an iterative process of adapting, extending and combining simpler models, with different models designed to address different questions. For example, asking when war will break out would require a different model from asking how long war will last. The process is encapsulated by the ABC of modelling in Table 1.1.

Needless to say, central to this approach is a suite of simpler models on which to build. So without further ado, let us start to develop them—beginning with Lecture 2.