9 War and Power: A Choice-Theoretic Model

So far we have not addressed the strategic issue of whether to go to war. Aspects of this issue can be analyzed with a choice-theoretic model, in the sense defined at the end of Lecture 8. We exemplify this approach with a model developed by Kim and Powell (1992), which explores the question of how relative power affects war decisions. Through this model, we also exemplify how nations' preferences over outcomes of war can be incorporated.

Nations rise and fall in power. A power transition between two nations is said to occur when a once dominant but now declining nation is overtaken by a nation whose power is on the rise—a challenger. It appears that such shifts in relative power sometimes lead to war, and at other times do not. So the question immediately arises: when do power shifts lead to war? This is the question that Kim and Powell set out to address with their choice-theoretic model, whose elements we now describe.

It will be convenient to refer to the challenger or rising power as Nation 1, and to the incumbent or declining power as Nation 2. We consider the change in their relative power over a transition period that extends from time $t = -1$, when Nation 2 is assumed to have greater power, to time $t = 1$, when Nation 2 is assumed to have greater power, with the power transition—equality of power—occurring at time $t = 0$. The challenger's rising power causes both nations to consider whether to go to war, and if so when—although war breaks out only if both nations are willing to fight. Specifically, for all $t \in [-1, 1]$, both nations consider whether to go to war at time t, but if neither has opted for war by time $t = 1$, then the transition is deemed to have happened peacefully.

Power is in essence about ability to get your way with others. Powerful nations get their way a lot in terms of, e.g., trade, territory and other resources, whereas weaker nations compromise or make concessions a lot. Power struggles between nations result in political outcomes—more or less temporary resolutions of disputes. We refer to the current such outcome as the status quo, and we assume that a better status quo for Nation 1 is automatically worse for Nation 2, and vice versa. Now, as we said in Lecture 1, the essence of modelling is abstraction, and in that regard it is convenient to idealize all possible status quos as numbers between 0 and 1. The left-hand extreme of the interval $[0, 1]$ represents the best possible outcome for Nation 2, in which it remains dominant and makes no concessions to Nation 1 in terms of trade or territory; it also represents the worst possible outcome for Nation 1. The right-hand extreme represents the best possible outcome for Nation 2, the terms that it would impose after total victory; it also represents the worst possible outcome for Nation 1. For definiteness, we can suppose that Nations 1 and 2 are in dispute over a territory, and that the status quo represents the proportion of this territory that Nation 1 controls.

The status quo can shift between the two extremes of 0 and 1 through diplomacy, but if it shifts through war, then the ultimate outcome is still just another status quo. So the outcome of a war that breaks out at time t can be idealized as a continuous random variable $X(t)$ with sample space $[0, 1]$. The set $\{X(t) | -1 \leq t \leq 1\}$ forms a continuous-time stochastic process on $[-1, 1]$, the interval corresponding to all possible status quos, and hence all possible outcomes of war. The larger the value of $x \in (0, 1)$, the more agreeable the outcome to Nation 1, the less agreeable to Nation 2. Let $U(x)$ denote the value or "utility" that Nation 1 derives per unit time from status quo x, and let $V(x)$ be the corresponding utility rate for Nation 2. Then

$$U'(x) \ > \ 0, \qquad V'(x) \ < \ 0 \tag{9.1}$$

for all $x \in (0, 1)$. No essential generality is lost by scaling utility so that

$$U(0) \ = \ 0 \ = \ V(1), \qquad U(1) \ = \ 1 \ = \ V(0). \tag{9.2}$$

Decision-makers are said to be (i) risk-averse, (ii) risk-neutral or (iii) risk-prone according to whether they (i) would prefer the status quo to an equal chance of gaining some ground or losing the same amount, (ii) would be indifferent between those options or (iii) would prefer the lottery. So Nation 1 is risk-averse if, for any status quo x and any amount h, $U(x) > U(x+h) \cdot \frac{1}{2} + U(x-h) \cdot \frac{1}{2} = U(x) + \frac{1}{2}h^2U''(x) + O(h^4)$, or if $U''(x) + O(h^2) < 0$. In the limit as $h \to 0$, we obtain $U''(x) < 0$. Correspondingly, Nation 1 is risk-neutral if $U''(x) = 0$ and risk-prone if $U''(x) > 0$, and likewise for Nation 2. In particular, (9.2) implies that Nation 1 is risk-neutral for

$$U(x) \ = \ x \tag{9.3}$$

(and likewise, Nation 2 is risk-neutral for $V(x) = 1 - x$). If we satisfy (9.1)–(9.2) by setting

$$U(x) \ = \ x^{\rho_1}, \qquad V(x) \ = \ (1-x)^{\rho_2} \tag{9.4}$$

then Nation i is risk-averse, risk-neutral or risk-prone according to whether $\rho_i < 1$, $\rho_i = 1$ or $\rho_i > 1$, for $i = 1, 2$. Moreover, risk aversion decreases with ρ_i or, equivalently, risk proneness increases with ρ_i.

Let $X(t)$ have cumulative distribution function F and probability density function f, that is, define

$$F(x,t) \ = \ \text{Prob}(X(t) \le x) \ = \ \int_0^x f(\xi, t)\, d\xi \tag{9.5}$$

on $[0, 1] \times [-1, 1]$ with

$$f(x,t) \ \ge \ 0 \tag{9.6}$$

on $(0, 1) \times [-1, 1]$, so that

$$f(x,t) \ = \ \frac{\partial F(x,t)}{\partial x} \tag{9.7}$$

on $(0, 1) \times [-1, 1]$; also $F(1, t) = 1$ or

$$\int_0^1 f(x,t)\, dx \ = \ 1. \tag{9.8}$$

An outcome favoring Nation 2 is likelier than one favoring Nation 1 for low values of t (near $t = -1$), whereas an outcome favoring Nation 1 is likelier for high values of t (near $t = 1$). Moreover, for $t = 0$ the distribution should be uniform (because at that time the nations equal one another in power). We therefore require

$$\frac{\partial f}{\partial x} \quad \begin{cases} < \ 0 & \text{if } \ t < 0 \\ = \ 0 & \text{if } \ t = 0 \\ > \ 0 & \text{if } \ t > 0 \end{cases} \tag{9.9}$$

in addition to (9.6)–(9.8). These constraints are most easily satisfied by choosing

$$f(x,t) \;=\; 1-\gamma t(1-2x) \tag{9.10}$$

where γ is the growth rate of Nation 1's power relative to that of Nation 2. We assume

$$0 \;<\; \gamma \;<\; 1, \tag{9.11}$$

so that (9.6) is invariably satisfied. From (9.4) and (9.10) the expected utility rates of a war that breaks out at time t can now be calculated. We obtain

$$u(t) \;\triangleq\; E[U(X(t))] \;=\; \int_0^1 U(x)f(x,t)\,dx \;=\; \frac{1}{1+\rho_1}+\frac{\gamma\rho_1 t}{(1+\rho_1)(2+\rho_1)} \tag{9.12}$$

for Nation 1 and

$$v(t) \;\triangleq\; E[V(X(t))] \;=\; \int_0^1 V(x)f(x,t)\,dx \;=\; \frac{1}{1+\rho_2}-\frac{\gamma\rho_2 t}{(1+\rho_2)(2+\rho_2)} \tag{9.13}$$

for Nation 2.

Now suppose that Nation 2 is challenged by Nation 1 at time t. If Nation 2 resists, then war breaks out, and its utility rate becomes $v(t)$ for the remainder of the transition period. So its total utility from $[t,1]$ is $(1-t)v(t)$. If, on the other hand, Nation 2 submits at time t, then—Kim and Powell (1992) assume—Nation 1 will challenge at all future times, and Nation 2 will submit to all of those challenges because its capabilities decline with time, and so if war is already unattractive at time t, then it will remain unattractive for all times $s>t$. Submitting means granting concessions, that is, agreeing to demands for shifting the status quo in Nation 1's favor. It is assumed that, for all $s>t$, the revised status quo to which Nation 2 agrees will be the same as the outcome of a war beginning at time s (although no such war breaks out). Hence for all $s\in(t,1)$, Nation 2's utility rate at time s will be $v(s)$; and so its total utility from the period $[t,1]$ is $\int_t^1 v(s)\,ds$. Let C be the cost of war. Then Nation 2 will resist when

$$(1-t)v(t)-C \;>\; \int_t^1 v(s)\,ds \tag{9.14}$$

and submit when the inequality is reversed. Substituting from (9.13), we find that Nation 2 will resist for $t<t_c$ and submit for $t>t_c$, where

$$t_c \;=\; 1-\sqrt{\frac{2(1+\rho_2)(2+\rho_2)}{\rho_2}}\sqrt{\frac{C}{\gamma}}. \tag{9.15}$$

It is straightforward to show that

$$\frac{\partial t_c}{\partial C} \;<\; 0, \qquad \frac{\partial t_c}{\partial \gamma} \;>\; 0, \tag{9.16}$$

so that Nation 2 will resist until later when war is less costly or its decline in relative power is more rapid, and that

$$\frac{\partial t_c}{\partial \rho_2} \text{ has the sign of } 2-{\rho_2}^2, \tag{9.17}$$

which is positive for $0 < \rho_2 < \sqrt{2}$ and negative thereafter. So decreasing risk aversion—or, equivalently, increasing risk proneness—delays the time when Nation 2 becomes indifferent between resisting and accepting a challenge from Nation 1, unless Nation 2 is so risk-prone that $\rho_2 > \sqrt{2}$. Moreover, since t_c has a maximum where $\rho_2 = \sqrt{2}$, we must have

$$t_c \;\le\; 1 - \sqrt{2}(1+\sqrt{2})\sqrt{\frac{C}{\gamma}}, \tag{9.18}$$

which exceeds -1 only if

$$C \;\le\; \frac{2\gamma}{(1+\sqrt{2})^2} \tag{9.19}$$

This is not, however, the conclusion reached in this regard by Kim and Powell (1992). Why? They chose to satisfy (9.1)–(9.2) not according to (9.4), but instead by setting

$$U(x) \;=\; x^{\rho_1}, \qquad V(x) \;=\; 1 - x^{1/\rho_2} \tag{9.20}$$

so that

$$v(t) \;=\; E[V(X(t))] \;=\; \frac{1}{1+\rho_2} - \frac{\gamma\rho_2 t}{(1+\rho_2)(1+2\rho_2)} \tag{9.21}$$

in place of (9.13) and

$$t_c \;=\; 1 - \sqrt{\frac{2(1+\rho_2)(1+2\rho_2)}{\rho_2}}\sqrt{\frac{C}{\gamma}} \tag{9.22}$$

in place of (9.15). Although (9.16) continues to hold, in place of (9.17) we find that

$$\frac{\partial t_c}{\partial \rho_2} \text{ has the sign of } 1 - 2{\rho_2}^2, \tag{9.23}$$

which is positive only for $0 < \rho_2 < 1/\sqrt{2}$ and negative thereafter.[1] So *in*creasing risk aversion—or, equivalently, decreasing risk proneness—delays the time when Nation 2 becomes indifferent between resisting and accepting a challenge from Nation 1, unless Nation 2 is so risk-averse that $\rho_2 < 1/\sqrt{2}$—a possibility that Kim and Powell (1992, p. 904) downplay on the grounds that it represents "extreme values."

Which is the better choice for U and V, (9.4) or (9.20)? It seems to me that if attitudes to risk are governed by the same underlying psychological process for both nations, then one would expect to have

$$V(x) \;=\; U(1-x) \text{ whenever } \rho_1 \;=\; \rho_2. \tag{9.24}$$

Because (9.4) satisfies (9.24) whereas (9.20) does not, I chose to depart from Kim and Powell (1992). On the other hand, (9.1), (9.2) and (9.24) would all be satisfied by

$$U(x) \;=\; 1 - (1-x)^{1/\rho_1}, \qquad V(x) \;=\; 1 - x^{1/\rho_2} \tag{9.25}$$

and would still lead to (9.22). So I don't know what to say, other than the obvious—how t_c depends on ρ_2 is sensitive to the form of V.

[1] Although (9.18)–(9.19) still hold.

The choice facing Nation 1 has now been reduced to whether it should challenge Nation 2 before the critical time, t_c. It will always challenge after t_c, because Nation 2 will submit to its demands without Nation 1 incurring any war cost. So let $t < t_c$, and consider Nation 1's options. If it does challenge, then war breaks out—because $t < t_c$ implies that Nation 2 will resist—and Nation 1's utility rate becomes $u(t)$ for the remainder of the transition period. So it c is the cost of war, then its net utility from the interval $[t, 1]$ will be $(1-t)u(t) - c$. If, on the other hand, Nation 1 does not challenge now, then it will accept the status quo until time t_c, when it knows that Nation 2 will submit to all future challenges without a fight. So, if the current status quo is x, then Nation 1's total utility from the interval $[t, t_c]$ is $(t_c - t)U(x)$; whereas, because the revised status quo to which Nation 2 agrees for all $s > t_c$ will be the same as the outcome of a war beginning at time s (though without such a war), Nation 1's total utility from the period $[t_c, 1]$ will be $\int_{t_c}^1 u(s)\,ds$. Thus the total utility of challenging—going to war—exceeds that of not challenging when

$$(1-t)u(t) - c \;>\; (t_c - t)U(x) + \int_{t_c}^1 u(s)\,ds \tag{9.26}$$

or, on using (9.4) and (9.12), when

$$\frac{(2t(1-t) - 1 + {t_c}^2)\gamma\rho_1}{2(1+\rho_1)(2+\rho_1)} \;+\; \left\{\frac{1}{1+\rho_1} - x^{\rho_1}\right\}(t_c - t) \;-\; c \;>\; 0. \tag{9.27}$$

Moreover, war is the more attractive, the greater the amount by which the left-hand side of this inequality exceeds zero. The left-hand side is clearly negative for $t = t_c$ and has a maximum, say $u_{\max}$, on $(-\infty, \infty)$ at $t = t_{\max}$, where

$$t_{\max} = \frac{1}{2}\left\{1 - \frac{(2+\rho_1)\{1-(1+\rho_1)x^{\rho_1}\}}{\gamma\rho_1}\right\}. \tag{9.28}$$

However, this maximum has no significance unless $t_{\max} < t_c$ and $u_{\max} > 0$; and whether these inequalities hold depends—in a complicated way—on the values of x, ρ_1, γ and c.

Kim and Powell proceed to investigate and draw conclusions, and if any of you is sufficiently interested, perhaps their subsequent analysis could form the basis of a more in-depth investigation leading to an end-of-term presentation. For the rest of us, however, their model has already served its dual purpose, namely, to exemplify how a choice-theoretic model can in principle be used to address a strategic issue, and to show how outcomes of war and nations' preferences over those outcomes can be idealized in terms of functions defined on $[0, 1]$ and satisfying (9.1)–(9.2).