11 Contest Models of War. The Paradox of Power

An important class of games has been developed to describe interactions in which two or more parties compete for a prize, expending effort in various ways to outcompete their rivals. The parties may range in size from individuals to nations. Both the interactions and the games used to describe them—in which each party is regarded as a player—have come to be known as contests (and no confusion ever seems to arise from using the same word for either, so we shall do likewise). In this context, war can be viewed as a possible contest outcome, and the model can be used to identify conditions that favor war or peace. The literature on such contests as games is now extensive, and it is common to begin with a general formalism in which the number of parties is an arbitrary finite number, say n, and only later to specialize to $n = 2$. Since $n = 2$ is the only value of n that we consider, however, we prefer to adopt a two-player formalism at the outset.

Consider, therefore, a pair of players engaged in a contest over a prize b, measured in suitable units of benefit. For $i = 1, 2$, let $U_i(b)$ denote the utility or value that Player i—or $\mathcal{P}_i$ for short—associates with the prize; let s_i denote the effort expended by $\mathcal{P}_i$ in competing for the prize; and let $C_i(s_i)$ denote the cost of its effort. We regard a player's effort as its strategy, hence our choice of notation; but in keeping with Lecture 10, it will be convenient to use the alternative notation

$$u \;=\; s_1, \qquad v \;=\; s_2 \tag{11.1}$$

as well. We use S_i to denote $\mathcal{P}_i$'s strategy set. Thus $u \in S_1$ is the effort expended by $\mathcal{P}_1$ in competing for the prize, and $v \in S_2$ is the corresponding effort for $\mathcal{P}_2$.

Let $p_i(u, v) \geq 0$ denote the probability that $\mathcal{P}_i$ wins the prize when efforts u, v are expended. Although it is possible that neither player wins the prize, so that $p_1(u, v) + p_2(u, v) < 1$, we shall assume for simplicity that one of them does, hence

$$p_1(u, v) + p_2(u, v) \;=\; 1. \tag{11.2}$$

So if the prize b is external to the dyad consisting of Players 1 and 2—that is, if it does not subtract from either player's resources—then $\mathcal{P}_i$'s payoff is the random variable

$$F_i \;=\; \begin{cases} U_i(b) \; - C_i(s_i) & \text{if } \mathcal{P}_i \text{ wins} \\ \quad 0 \quad\; - C_i(s_i) & \text{if } \mathcal{P}_i \text{ loses} \end{cases} \tag{11.3}$$

and $\mathcal{P}_i$'s reward from the strategy combination (u, v) is the expected value of F_i, which we denote by $f_i(u, v)$. That is, using E to denote expectation,

$$\begin{aligned} f_i(u, v) \;=\; \mathrm{E}[F_i] \;&=\; \{U_i(b) - C_i(s_i)\} \cdot \mathrm{Prob}(\mathcal{P}_i \text{ wins}) + \{0 - C_i(s_i)\} \cdot \mathrm{Prob}(\mathcal{P}_i \text{ loses}) \\ &=\; \{U_i(b) - C_i(s_i)\} \cdot p_i(u, v) + \{0 - C_i(s_i)\} \cdot \{1 - p_i(u, v)\} \\ &=\; p_i(u, v)\, U_i(b) - C_i(s_i) \end{aligned} \tag{11.4}$$

by (11.2) for all $(u, v) \in D$, where $D = S_1 \times S_2$ is the decision set. If, on the other hand, the prize is internal to the dyad, then (11.3) no longer holds, although the resulting rewards may still in effect be given by (11.4), as illustrated by Lecture 12. The above is anyhow the most general formalism that we consider.[1]

For the most part, however, a tractable model requires further specification of U_i, C_i and p_i, and in this respect we follow what is customary in the literature. First, we assume that the players are risk-neutral. Hence, by (9.3), we set

$$U_i(b) \;=\; b \tag{11.5}$$

(noting that utility can be scaled to between 0 and 1 without loss of generality). Second, we assume that

$$C_i(s_i) \;=\; \kappa\, s_i \tag{11.6}$$

where κ (>0) denotes the marginal cost of effort, assumed the same for both players. Third, we need an explicit form for the function p_i, which is known in the literature as a "contest success function" (Hirshleifer, 1989) or "conflict success function" (Anderton and Carter, 2009, p. 246), and either way is CSF for short. Two forms prevail. The first form has both a common one-parameter version defined on $(0,\infty)\times(0,\infty)$ by

$$p_i(s_1,s_2) \;=\; \frac{s_i^\gamma}{s_1^\gamma+s_2^\gamma} \tag{11.7}$$

and a less common two-parameter version defined on $(0,\infty)\times(0,\infty)$ by

$$p_1 \;=\; \frac{\lambda s_1^\gamma}{\lambda s_1^\gamma+s_2^\gamma}, \qquad p_2 \;=\; \frac{s_2^\gamma}{\lambda s_1^\gamma+s_2^\gamma} \tag{11.8}$$

(where, of course, (11.7) is the special case of (11.8) for which $\lambda=1$). Note that this CSF is homogeneous of degree zero, that is,

$$p_i(ks_1,ks_2) \;=\; k^0\, p_i(s_1,s_2) \;=\; p_i(s_1,s_2) \tag{11.9}$$

for any scale factor $1/k$; the CSF is said to be in ratio form, because (11.8) implies that

$$p_1 \;=\; \frac{\lambda(s_1/s_2)^\gamma}{\lambda(s_1/s_2)^\gamma+1}, \qquad p_2 \;=\; \frac{1}{\lambda(s_1/s_2)^\gamma+1} \tag{11.10}$$

are functions of s_1/s_2 alone. The second CSF is defined on $[0,\infty)\times[0,\infty)$ by

$$p_i(s_1,s_2) \;=\; \frac{e^{\gamma s_i}}{e^{\gamma s_1}+e^{\gamma s_2}} \tag{11.11}$$

and is said to be in difference form, because (11.11) implies that

$$p_1 \;=\; \frac{1}{1+e^{-\gamma(s_1-s_2)}}, \qquad p_2 \;=\; \frac{1}{1+e^{\gamma(s_1-s_2)}} \tag{11.12}$$

are functions of s_1-s_2 alone. In either case, γ is a sensitivity parameter, which can be interpreted as a "decisiveness coefficient" (Anderton and Carter, 2009, p. 247): for (11.7)

[1] It corresponds to the special case of Konrad (2009, p. 2) for which $n=2$.

or (11.8) it measures sensitivity to effort ratio, whereas for (11.11) it measures sensitivity to effort difference. To interpret λ, we note from (11.8) or (11.10) that $p_1/p_2 = \lambda$ when $s_1 = s_2$. Thus λ measures the extent to which the outcome is biased towards $\mathcal{P}_1$ when efforts are equal. It can be therefore interpreted as relative measure of Player 1's fighting skills or some other advantage.[2]

The definition of the CSF in ratio form is frequently extended to $[0, \infty) \times [0, \infty)$ by stipulating that $p_1 = p_2 = \frac{1}{2}$ for $s_1 = s_2 = 0$ (e.g., Beviá and Corchón, 2010, p. 472). However, it may not be appropriate to assume that the outcome is decided by pure lottery in the absence of effort on either side; and in any event, the CSF would remain discontinuous at $(0, 0)$, which is clearly best avoided. We sidestep this issue here by simply assuming that there is positive effort on both sides (unless there is no contest).

Let us now follow Anderton and Carter (2009, p. 249) in assuming that Players 1 and 2 are nations in dispute over an external resource of fixed value b. For $i = 1, 2$, let Nation i control resources of value b_i, which they are free to expend on producing military goods; without loss of generality, we assume that

$$b_1 \;\geq\; b_2. \tag{11.13}$$

(So we can think of Nation 1 as the rich nation and Nation 2 as the poor one, at least when the inequality is strict.) In this context, the contest efforts u and v become the efforts that Nations 1 and 2, respectively, expend on war. Both u and v are positive (unless there is no war). Because Nation i's war effort comes out of the resources that it controls, which total b_i, we also require $C_i(s_i) = \kappa s_i \leq b_i$ for both i. So

$$S_i \;=\; \big(0, b_i/\kappa\big] \tag{11.14}$$

is $\mathcal{P}_i$'s strategy set and

$$D \;=\; \big(0, b_1/\kappa\big] \times \big(0, b_2/\kappa\big] \tag{11.15}$$

is their joint decision set.

For the sake of simplicity, we now assume that $\gamma = 1$ in (11.8), so that

$$p_1(u, v) \;=\; \frac{\lambda u}{\lambda u + v}, \qquad p_2(u, v) \;=\; \frac{v}{\lambda u + v} \tag{11.16}$$

by (11.1). So, on using (11.4)–(11.6), the rewards are given by

$$\left.\begin{aligned} f_1(u, v) \;&=\; \frac{\lambda u b}{\lambda u + v} - \kappa u \\ f_2(u, v) \;&=\; \frac{v b}{\lambda u + v} - \kappa v \end{aligned}\right\} \quad \text{for } (u, v) \in D. \tag{11.17}$$

[2]For example, it is described as the "relative effectiveness" of $\mathcal{P}_1$'s "military goods" by Anderton and Carter (2009, p. 247). These authors also introduce a two-parameter version of (11.11), namely,

$$p_1 \;=\; \frac{e^{\lambda\gamma s_1}}{e^{\lambda\gamma s_1} + e^{\gamma s_2}}, \qquad p_2 \;=\; \frac{e^{\gamma s_2}}{e^{\lambda\gamma s_1} + e^{\gamma s_2}}$$

and claim that it "depends on the difference" $s_1 - s_2$ (Anderton and Carter, 2009, p. 248); however, it actually depends on $\lambda s_1 - s_2$, which is not quite the same thing.

We note in passing that the parameter κ could now be scaled out of the problem by scaling effort with respect to $\frac{1}{\kappa}$. If we were to define new variables by $\tilde{u} = \kappa u$, $\tilde{v} = \kappa v$, thus measuring effort in units of cost, then (11.17) would become

$$\begin{aligned} f_1(\tilde{u},\tilde{v}) &= \frac{\lambda \tilde{u} b}{\lambda \tilde{u} + \tilde{v}} - \tilde{u} \\ f_2(\tilde{u},\tilde{v}) &= \frac{\tilde{v} b}{\lambda \tilde{u} + \tilde{v}} - \tilde{v} \end{aligned} \qquad \text{for } (u,v) \in (0,b_1] \times (0,b_2]. \tag{11.18}$$

It is merely convenient not to do so here—although we do measure war effort in units of cost in Lecture 12. End of digression.

The reaction sets[3] are now readily calculated. Let us first define

$$\hat{u}(v) = \sqrt{\frac{bv}{\kappa\lambda}} - \frac{v}{\lambda}, \qquad \hat{v}(u) = \sqrt{\frac{b\lambda u}{\kappa}} - \lambda u. \tag{11.19}$$

Then because

$$\frac{\partial f_1}{\partial u} = \frac{\lambda b v}{(\lambda u + v)^2} - \kappa, \tag{11.20}$$

implying $\partial f_1/\partial u|_{u=0} = \lambda b/v - \kappa$ and $\partial^2 f_1/\partial u^2 = -2\lambda^2 bv/(\lambda u + v)^3 < 0$, we see that when $v \geq \lambda b/k$, f_1 is strictly decreasing with respect to u for all $u > 0$, and hence has its maximum with respect to u where $u = 0$; whereas when $0 < v < \lambda b/k$, f_1 increases from 0 as $u \to 0$ to a maximum of

$$f_1(\hat{u}(v), v) = \left(\sqrt{b} - \sqrt{\frac{\kappa v}{\lambda}}\right)^2 \tag{11.21}$$

at $u = \hat{u}(v)$ before decreasing towards 0 again as $u \to b/k - v/\lambda$. Moreover, because (11.19) implies

$$\hat{u}'(v) = \frac{1}{2}\sqrt{\frac{b}{\lambda\kappa v}} - \frac{1}{\lambda}, \qquad \hat{u}''(v) = -\frac{1}{4v}\sqrt{\frac{b}{\lambda\kappa v}} < 0, \tag{11.22}$$

we see that $\hat{u}(v)$ increases from $\hat{u}(0) = 0$ to $\hat{u}(\frac{\lambda b}{4\kappa}) = \frac{b}{4\kappa}$ on $[0, \frac{\lambda b}{4\kappa}]$ before decreasing on $[\frac{\lambda b}{4\kappa}, \frac{\lambda b}{\kappa}]$ to $\hat{u}(\frac{\lambda b}{\kappa}) = 0$, implying that $\hat{u}(v)$ has maximum $\frac{b}{4\kappa}$ on $[0, \frac{\lambda b}{\kappa}]$, as indicated by the vertical dashed line in Figure 11.1. So $\hat{u}(v)$ is certainly the best response to v when $\frac{b}{4\kappa} \leq b_1/\kappa$ or $\frac{b}{4} \leq b_1$, ensuring $\hat{u}(v) \in S_1$ for all $v \in S_2$ by (11.14); then

$$\mathcal{B}_1(v) = \hat{u}(v) \quad \text{for all } v \in S_2, \tag{11.23}$$

as illustrated by Figure 11.1. A similar analysis shows that when $u \geq b/(\lambda k)$, f_2 is strictly decreasing with respect to v for all $v > 0$, and hence has its maximum with respect to v

[3] Defined, you will recall, by (10.16) as

$$\begin{aligned} R_1 &= \{(u,v) \in D \mid f_1(u,v) = \max_{\overline{u}} f_1(\overline{u}, v)\} = \{(u,v) \in D \mid u = \mathcal{B}_1(v)\} \\ R_2 &= \{(u,v) \in D \mid f_2(u,v) = \max_{\overline{v}} f_2(u, \overline{v})\} = \{(u,v) \in D \mid v = \mathcal{B}_2(u)\} \end{aligned}$$

where $v = 0$; whereas when $0 < u < b/(\lambda k)$, f_2 has its maximum with respect to v on $[0, \frac{b}{\kappa} - \lambda u]$ at $v = \hat{v}(u)$ defined by (11.19), the maximum being

$$f_2(u, \hat{v}(u)) \;=\; \left(\sqrt{b} - \sqrt{\lambda\kappa v}\,\right)^2. \tag{11.24}$$

Moreover, $\hat{v}(u)$ has its maximum $\hat{v}(\frac{b}{4\kappa\lambda}) = \frac{b}{4\kappa}$ on $[0, \frac{b}{\lambda\kappa}]$, as indicated by the horizontal dashed line in Figure 11.1. So (11.23) has the companion result that

$$\mathcal{B}_2(u) \;=\; \hat{v}(u) \quad \text{for all } u \in S_1 \tag{11.25}$$

when $\frac{b}{4\kappa} \leq b_2/\kappa$ or $\frac{b}{4} \leq b_2$, again as illustrated by Figure 11.1. The Nash equilibrium occurs where R_1 and R_2 intersect at $(u^*, v^*) \in D$, that is, where

$$\hat{u}(v^*) \;=\; u^* \quad \text{and} \hat{v}(u^*) \;=\; v^* \tag{11.26}$$

or

$$u^* \;=\; v^* \;=\; \frac{\lambda b}{(1+\lambda)^2\,\kappa} \tag{11.27}$$

by (11.19).[4] By (11.17), this equilibrium yields reward

$$w_1 \;=\; f_1(u^*, v^*) \;=\; \left(\frac{\lambda}{1+\lambda}\right)^2 b \tag{11.28}$$

to Player 1 and

$$w_2 \;=\; f_2(u^*, v^*) \;=\; \frac{b}{(1+\lambda)^2} \;=\; \frac{f_1(u^*, v^*)}{\lambda^2} \tag{11.29}$$

to Player 2.

This result illustrates the so called "paradox of power" (Hirshleifer, 1991; Anderton and Carter, 2009, p. 253) for $\lambda \leq 1$. If $\lambda = 1$, so that neither nation has greater fighting skills than the other, then both nations obtain the same reward, even though Nation 2 may be very much poorer than Nation 1, in the sense that b_2/b_1 may be very much less than 1. If $\lambda < 1$, so that Nation 2 has greater fighting skills than Nation 1, then Nation 2's reward will be greater than Nation 1's, no matter how much poorer Nation 2 might be. At first it does seem paradoxical that the Nash equilibrium is independent of b_1 and b_2, but a little reflection reveals that we have virtually assumed that it would be, since λ has been assumed independent of b_1 and b_2—we therefore allow a much poorer country to be very much better at fighting, no matter how much poorer they are.

[4]By (11.19), (11.26) implies

$$\sqrt{\frac{bv^*}{\kappa\lambda}} - \frac{v^*}{\lambda} \;=\; u^* \quad \text{and} \quad \sqrt{\frac{b\lambda u^*}{\kappa}} - \lambda u^* \;=\; v^*,$$

from which

$$\sqrt{\frac{b\lambda v^*}{\kappa}} \;=\; \lambda u^* + v^* \;=\; \sqrt{\frac{b\lambda u^*}{\kappa}}$$

implies $u^* = v^*$. Substituting back, we obtain $(1+\lambda)\sqrt{v^*} = \sqrt{b\lambda/\kappa}$, which reduces to (11.27).

Figure 11.1: The reaction sets R_1 (solid green), R_2 (solid red) and Nash equilibrium (blue dot) for $\frac{1}{4}b < b_2 < b_1$ and $\frac{1}{4} < \lambda < 1$. The decision set $D = S_1 \times S_2$ is shaded.

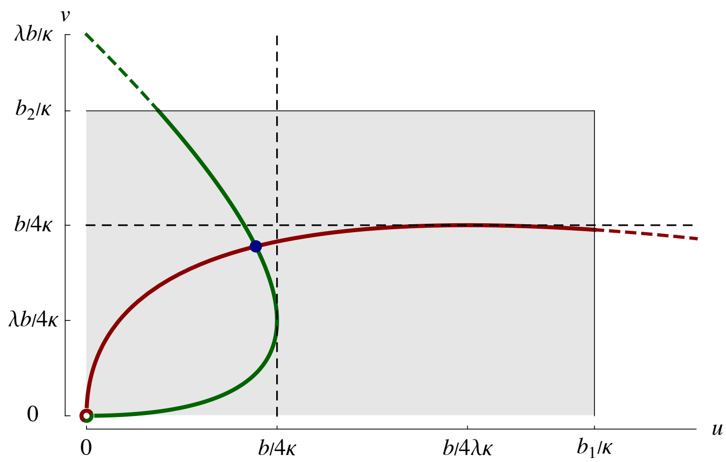
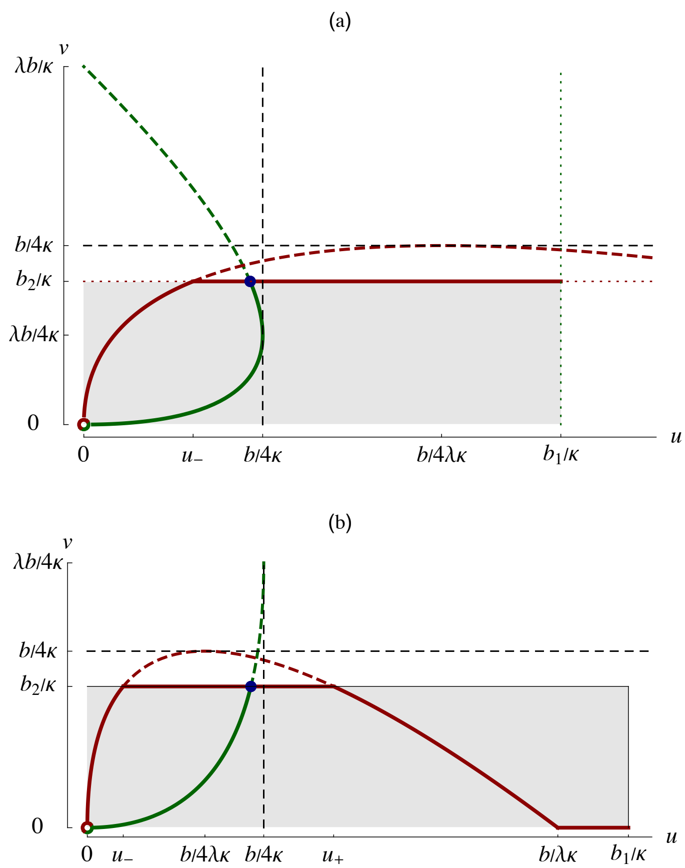


Figure 11.2: The reaction sets R_1 (solid green), R_2 (solid red) and Nash equilibrium (blue dot) for $b_2 < \frac{1}{4}b < b_1$ for (a) $\frac{1}{4} < \lambda < 1$ and (b) $1 < \lambda < 4$. The decision set $D = S_1 \times S_2$ is shaded.



On the other hand, Figure 11.1 is by no means the whole story, because we have also assumed $\frac{1}{4}b \le b_2$ (and hence, by (11.13), that $\frac{1}{4}b \le b_1$): the disputed resource is worth less than four times the resources that the poorer nation controls exclusively. For $i = 1, 2$, let us define

$$\beta_i \;=\; \frac{b_i}{b} \tag{11.30}$$

so that $\frac{1}{4}b \le b_2$ becomes $\beta_2 \ge \frac{1}{4}$, and assume instead that

$$\beta_2 \;<\; \tfrac{1}{4} \tag{11.31}$$

or $\frac{1}{4}b > b_2$. Then $\hat{v}(u) > b_2/\kappa$ for $u \in (u_-, u_+)$, where we define

$$u_\pm \;=\; \tfrac{1}{2\lambda\kappa}\big\{b - 2b_2 \pm \sqrt{b(b-4b_2)}\big\}. \tag{11.32}$$

Because $u \in (u_-, u_+)$ implies $\partial f_2/\partial v > 0$ for all $v \in (0, b_2/\kappa)$, the best reply to all $u \in (u_-, u_+)$ becomes $v = b_2/\kappa$. So

$$\mathcal{B}_2(u) \;=\; \begin{cases} \hat{v}(u) & \text{if } 0 < u < u_- \\ b_2/\kappa & \text{if } u_- \le u \le b_1/\kappa \end{cases} \qquad \text{when } u_+ \ge b_1/\kappa \tag{11.33}$$

as illustrated by Figure 11.2(a), whereas

$$\mathcal{B}_2(u) \;=\; \begin{cases} \hat{v}(u) & \text{if } 0 < u < u_- \\ b_2/\kappa & \text{if } u_- \le u \le u_+ \\ \hat{v}(u) & \text{if } u_+ < u \le b_1/\kappa \end{cases} \qquad \text{when } u_+ < b_1/\kappa < \tfrac{b}{\lambda\kappa} \tag{11.34}$$

and

$$\mathcal{B}_2(u) \;=\; \begin{cases} \hat{v}(u) & \text{if } 0 < u < u_- \\ b_2/\kappa & \text{if } u_- \le u \le u_+ \\ \hat{v}(u) & \text{if } u_+ < u < \tfrac{b}{\lambda\kappa} \\ 0 & \text{if } \tfrac{b}{\lambda\kappa} \le u \le b_1/\kappa \end{cases} \qquad \text{when } b_1/\kappa > \tfrac{b}{\lambda\kappa} \tag{11.35}$$

as illustrated by Figure 11.2(b). Now R_1 and R_2 intersect where $v = v^* = b_2/\kappa$ and $u = u^* = \hat{u}(v^*) = \hat{u}(b_2/\kappa)$. Accordingly, by (11.19), we obtain

$$u^* \;=\; \frac{1}{\kappa}\left\{\sqrt{\frac{b_2 b}{\lambda}} - \frac{b_2}{\lambda}\right\}, \qquad v^* \;=\; \frac{b_2}{\kappa} \tag{11.36}$$

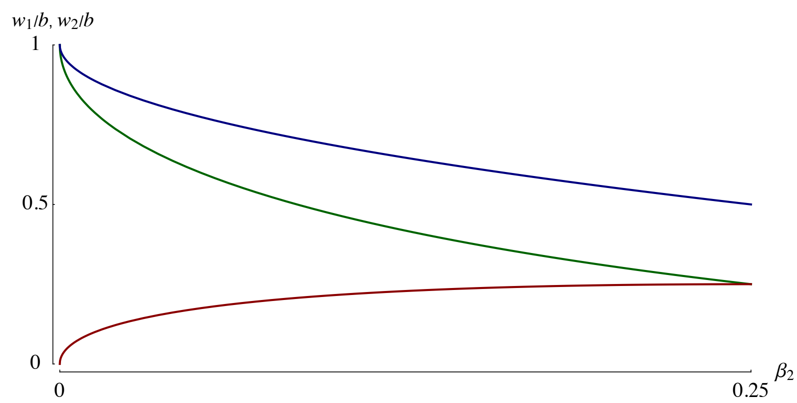
at the Nash equilibrium. By (11.17), this equilibrium yields reward

$$w_1 \;=\; f_1(u^*, v^*) \;=\; \left(\sqrt{\frac{\lambda b}{b_2}} - 1\right)^2 \frac{b_2}{\lambda} \;=\; \left(1 - \sqrt{\frac{\beta_2}{\lambda}}\right)^2 b \tag{11.37}$$

to Player 1 and

$$w_2 \;=\; f_2(u^*, v^*) \;=\; \left(\sqrt{\frac{b}{\lambda b_2}} - 1\right) b_2 \;=\; \left(\sqrt{\frac{1}{\lambda\beta_2}} - 1\right)\beta_2 b \tag{11.38}$$

Figure 11.3: Proportions of the prize b obtained by Nation 1 (green) and Nation 2(red), together with the proportion of the prize that is won by either side (blue) for the Nash equilibrium in Figure 11.2.



to Player 2, where β_2 is defined by (11.30). The proportions of the prize obtained at equilibrium are plotted in Figure 11.3 for $\lambda = 1$. We see that the rewards are no longer equal, except in the limit as $\beta_2 \to \frac{1}{4}$.

Figure 11.2 assumes that $\frac{1}{4}b < b_1$, but a further possibility is that $\frac{1}{4}b > b_1 \geq b_2$, as in Figure 11.4. Now (11.33) continues to hold, but (11.23) is replaced by

$$\mathcal{B}_1(v) \;=\; \begin{cases} \hat{u}(v) & \text{if } 0 < v < v_- \\ b_1/\kappa & \text{if } v_- \leq v \leq b_2/\kappa \end{cases} \tag{11.39}$$

where

$$v_\pm \;=\; \tfrac{\lambda}{2\kappa}\big\{b - 2b_1 \pm \sqrt{b(b - 4b_1)}\big\} \tag{11.40}$$

and R_1, R_2 intersect on the boundary of D, so that

$$u^* \;=\; \frac{b_1}{\kappa}, \qquad v^* \;=\; \frac{b_2}{\kappa} \tag{11.41}$$

at the Nash equilibrium, as illustrated by Figure 11.4. By (11.17), this equilibrium yields

$$w_1 \;=\; f_1(u^*, v^*) \;=\; \left(\frac{\lambda b}{\lambda b_1 + b_2} - 1\right) b_1 \;=\; \frac{\{\lambda(1 - \beta_1) - \beta_2\}\beta_1 b}{\lambda\beta_1 + \beta_2} \tag{11.42}$$

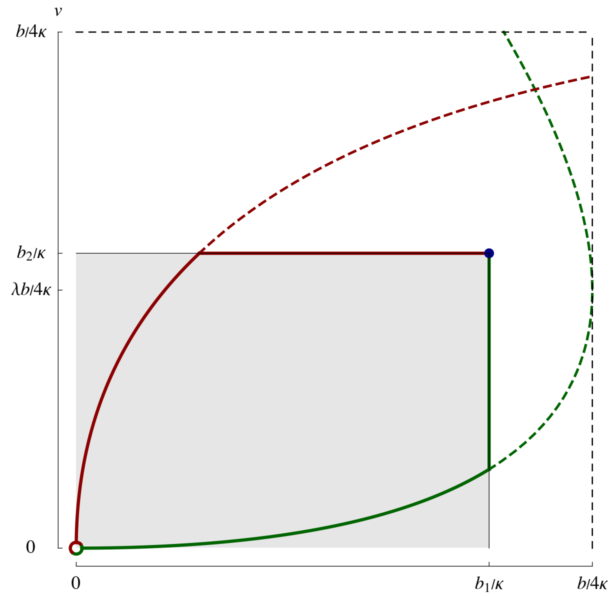
to Player 1 and

$$w_2 \;=\; f_2(u^*, v^*) \;=\; \left(\frac{b}{\lambda b_1 + b_2} - 1\right) b_2 \;=\; \frac{\{1 - \lambda\beta_1 - \beta_2\}\beta_2 b}{\lambda\beta_1 + \beta_2} \tag{11.43}$$

to Player 2, on using (11.30). So

$$\frac{w_2}{w_1} \;=\; \frac{\{1 - \lambda\beta_1 - \beta_2\}\beta_2}{\{\lambda(1 - \beta_1) - \beta_2\}\beta_1} \;=\; \frac{\beta_2}{\beta_1} \tag{11.44}$$

Figure 11.4: The reaction sets R_1 (solid green), R_2 (solid red) and Nash equilibrium (blue dot) for $b_2 < b_1 < \frac{1}{4}b$ and $\frac{1}{4} < \lambda < 1$. The decision set $D = S_1 \times S_2$ is shaded.



when $\lambda = 1$ (which is scarcely paradoxical).

This is still by no means the end of the story, partly because other configurations for R_1 and R_2 are possible even though no new types of Nash equilibria are thereby introduced—for example, the interior Nash equilibrium seen in Figure 11.1 may also arise with both R_1 and R_2 having a boundary segment when λ is sufficiently small, as illustrated by Figure 11.5—but mainly because we have presupposed a war. Yet war may be in neither nation's interest. In this regard, let δ denote the proportion of any external prize that war destroys. Then the original value of that prize is

$$B \;=\; \frac{b}{\delta}. \tag{11.45}$$

Let η_i denote the proportion of the prize that Nation i can obtain through diplomatic negotiations, where $\eta_1 + \eta_2 \leq 1$. Then both nations should prefer peace—that is, zero war effort—to war when

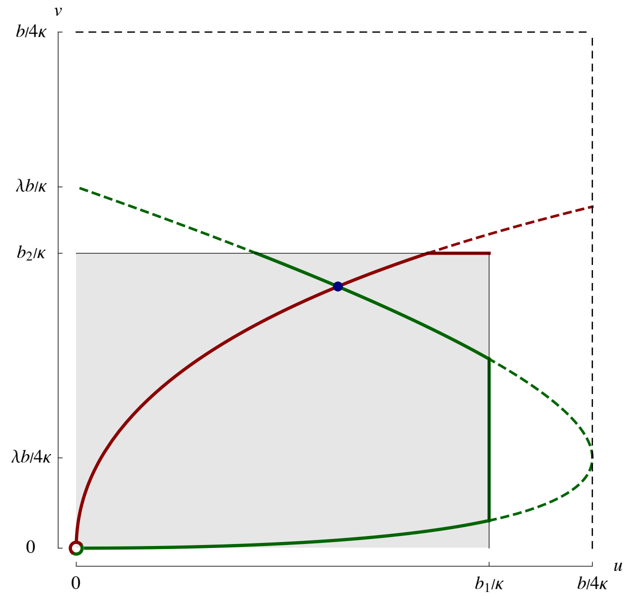
$$\eta_i B \;\geq\; w_i \tag{11.46}$$

for $i = 1, 2$. Here we assume that if the same amount of resources can be obtained either through war or by peaceful means, then the second option will always be preferred to the first—hence weak, as opposed to strong, inequality in (11.46).

At least three issues now arise more or less at once. The first is how to determine η_1 and η_2. The second is that zero war effort on both sides is not a Nash equilibrium, because $(0,0) \notin R_1 \cap R_2$: zero war effort is not self-enforcing, even if both sides prefer it.[5]

[5]Because $f_1(u,0) = b - \kappa u$ for $u \neq 0$ by (11.17), the best reply to $v = 0$ for $\mathcal{P}_1$ is any small positive effort, say ϵ, which secures virtually all of the prize b and guarantees that $f_1(\epsilon, 0) > f_1(0,0)$, regardless of whether we set $p_i(0,0) = \frac{1}{2}$ (implying $f_1(0,0) = \frac{1}{2}b$) or $p_i(0,0) = 0$; and likewise for $\mathcal{P}_2$.

Figure 11.5: The reaction sets R_1 (solid green), R_2 (solid red) and Nash equilibrium (blue dot) for $b_2 < b_1 < \frac{1}{4}b$ and $\lambda < \frac{1}{4}$. The decision set $D = S_1 \times S_2$ is shaded.



Accordingly, the third issue is how to enforce a peace agreement.

For now, however, we put these issues aside.