

Coordinate Systems for Conformal Cerebellar Flat Maps

Monica K. Hurdal*, Ken Stephenson†, Phil Bowers*,
De Witt Sumners*, David A. Rottenberg‡§

*Department of Mathematics, Florida State University, Tallahassee, U.S.A.

†Department of Mathematics, University of Tennessee, Knoxville, U.S.A.

‡PET Imaging Center, VA Medical Center, Minneapolis, MN, U.S.A.

§Departments of Neurology and Radiology, University of Minnesota, Minneapolis, MN, U.S.A.

We have generated quasi-conformal flat maps of the human cerebellum using circle packings (1,2,3,4). Conformal maps, which preserve angular proportion and angle direction between curves, are mathematically unique and carry valuable geometric structure. The quasi-conformal maps we obtain via circle packing approximate conformal maps and share many of their mathematical advantages. In particular, they can be created on the Euclidean and hyperbolic planes as well as on a sphere. The origin of hyperbolic maps can be interactively transformed so that user-defined anatomical or functional landmarks can serve as the map focus. One can impose canonical surface-based coordinate systems on these flat maps by specifying anatomical or functional landmarks: two for hyperbolic coordinates, three for spherical coordinates. We describe the use of anatomical landmarks for imposing canonical coordinate systems on cerebellar flat maps, and for defining anatomical features and localizing functional activations.

Methods

Quasi-conformal flat maps of the human cerebellum (1,4) were created from a high-resolution T1-weighted MRI volume (5). A topologically correct surface was produced from a cerebellar volume defined by a plane parallel to the posterior commissure-obex line and orthogonal to a plane passing through the vermal midline. Our quasi-conformal flattening procedure (2,3) was applied to this surface to produce flat maps; the cortical surface was parcellated according to (6), and activations produced by a target interception task were imposed. Eleven readily identifiable anatomical landmarks, e.g., the apex of the fourth ventricle and the rostral-most tip of the lingula, were defined. For the hyperbolic coordinate system, one landmark was used as the map center and a second to specify the map orientation. We also used a polar coordinate system on the hyperbolic flat map, with equiangular grid lines radiating from the map focus and equidistant circles surrounding the focus. For the spherical coordinate system, one landmark was used as the north pole, one as the south pole, and a third as a distinguished equatorial point. We used the usual latitude and longitude coordinates, similar to (7).

Results and Conclusions

A major benefit of (quasi-)conformal flat maps is that they can be used to generate canonical surface-based coordinate systems for a cortical surface. Since our flat maps to the hyperbolic plane and the sphere are 1-1 and onto, the coordinate systems produced by these flat maps can be used to quantitatively localize structure and function. Furthermore, these flat maps are mathematically unique. Different choices of landmarks and coordinate systems will increase the utility of flat maps for studying functional imaging data. We believe that the use of cerebellar flat maps and canonical coordinate systems based on reproducible anatomical landmarks will improve our ability to localize functional activity on the cerebellar cortex and to quantify anatomical and functional differences between individual subjects and groups of subjects.

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