# Quantitative Evaluation of Three Cortical Surface Flattening Methods


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## Abstract

During the past decade several computational approaches have been proposed for mapping highly convoluted surfaces of the human brain to simpler geometric objects such as a sphere or a plane. We report the results of a quantitive comparison of FreeSurfer, CirclePack, and LSCM with respect to measurements of geometric distortion and computational speed. Our results indicate that FreeSurfer performs best with respect to a *global* measurement of metric distortion whereas LSCM is computationally much more efficient and outperforms FreeSurfer and CirclePack with respect to angular distortion and a *local* measurement of metric distortion.


## Methods

Let $\mathcal{K}$ be a simply-connected triangulated cortical surface $\{\{\mathbf{v}_i\}_{i=1}^n, \mathcal{T} = \{T_i = (\mathbf{v}_{i_1}, \mathbf{v}_{i_2}, \mathbf{v}_{i_3})\}_{i=1}^m\}$ where $\{\mathbf{v}_i\}_{i=1}^n$ is a set of $n$ vertices with $n \geq 3$ and $\mathcal{T}$ is a set of $m$ triangles consisting of triples of vertices and let $\mathcal{U}$ represent its flat mapping function. Assume that $\mathcal{U}$ is linear on each triangle $T_i$. Denote by $A(T)$ the oriented area of the triangle $T$ and by $d_{i,j}$ and $d^{\mathcal{U}}_{i,j}$ the geodesic distances between the vertex $\mathbf{v}_i$ and $\mathbf{v}_j$ on the original cortical surface $\mathcal{K}$ and its flat map $\mathcal{U}(\mathcal{K})$ respectively. Denote by $T_i^{\mathcal{U}} = \mathcal{U}(T_i)$ the map of the triangle $T_i$.

### FreeSurfer

FreeSurfer [1] is a popular software package for cortical surface flattening that explicitly minimizes the metric distortion of the flattened cortical surface. Define the mean-squared energy functionals related to metric information and oriented area respectively such as

$$\mathbf{J}_d = \frac{1}{4n}\sum_{i=1}^{n}\sum_{j\in N_i}(d^{\mathcal{U}}_{i,j} - d_{i,j})^2, \quad \mathbf{J}_a = \frac{1}{2m}\sum_{i=1}^{m} P(A(T_i^{\mathcal{U}}))(A(T_i^{\mathcal{U}})) - A(T_i))^2$$

where $N_i$ denotes the set of vertices which are pre-defined neighbors of the vertex $i$ and $P(A(T_i^{\mathcal{U}})) = 1$ if $A(T_i^{\mathcal{U}}) > 0$, otherwise $P(A(T_i^{\mathcal{U}})) = 0$. Then, the complete functional becomes

$$\mathbf{J} = \lambda_d \mathbf{J}_d + \lambda_a \mathbf{J}_a \tag{1}$$

where $\lambda_a$ and $\lambda_d$ reflect the relative importance of unfolding versus the minimization of metric distortion.

- Initially $\lambda_a$ takes much larger values than $\lambda_d$ and gradually decreases over times as the surface is successfully unfolded.
- To generate a spherical map the inflated cortical surface is projected onto the sphere by moving each vertex to the closest point on the sphere; the energy functional is again minimized to reduce the metric distortion and remove any folds introduced by the projection process.

### CirclePack

CirclePack [2] is a quasi-conformal flattening method and depends solely on the combinatorics of the original cortical surface. It can be described as follows: for a collection of circles $\mathbf{C}_{\mathcal{K}} = \{C(\mathbf{v}_i)\}_{i=1}^n$ in the plane, one circle for each vertex $\mathbf{v}_i$, has the property that $C(\mathbf{v}_i)$ and $C(\mathbf{v}_j)$ are tangent whenever $\mathbf{v}_i$ and $\mathbf{v}_j$ form an edge of $\mathcal{K}$.

- The *Circle Packing Theorem* states that given any disk triangulation $\mathcal{K}$ and any assignment of positive number $r_1, r_2, \cdots, r_{n_b}$ to the whole boundary vertices $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_{n_b}$ of $\mathcal{K}$, there is a unique circle packing (up to Euclidean isomotry) in the plane with boundary circle $C(\mathbf{v}_i)$ having the radius $r_i$.
- It is noted that if $\mathcal{K}$ consists only of equilateral triangles then the map of CirclePack converges to a conformal map.
- Spherical maps can be generated by the stereographic projection of hyperbolic maps and normalized by a Mobius transformation to minimize the metric distortion among the automorphism group.

### LSCM

LSCM [3] is a conformal flattenning method. Suppose that $\mathcal{K}$ is a topological disk. When restricting $\mathcal{U}$ on one of the triangles of $\mathcal{T}$, say $T$, the Cauchy-Reimann equation states that $\mathcal{U}$ is conformal on $T$ if and only if the following equality holds true on $T$: $\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y} = 0$. This conformal criterion generally cannot be strictly satisfied on the whole $\mathcal{K}$, so minimization of violation of this condition is used to construct the quasi-conformal map in the least square sense:

$$\min_{\mathcal{U}} \; C(\mathcal{K}) = \sum_{T\in\mathcal{T}} \int_T \left|\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y}\right|^2 dA = \sum_{T\in\mathcal{T}} \left|\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y}\right|^2 A(T)$$

Suppose that $\mathbf{u}_i = \mathcal{U}(\mathbf{v}_i)$ for $i = 1, \cdots, n$, then $C(\mathcal{K})$ can be written in the *quadratic form* such as

$$\min_{\mathcal{U}} C(\mathcal{K}) = \mathbf{u}^* M^* M \mathbf{u} \tag{2}$$

- So that the minimization problem (2) has a unique and non-trivial solution, some $\mathbf{u}_i'$s must be pre-defined. The spherical map is again obtained by the stereographic projection and the normalization process.
- *Adaptive Weighted LSCM* approach:

$$\min_{\mathcal{U}} \; C(\mathcal{K}) = \sum_{T\in\mathcal{T}} \alpha(T)\left|\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y}\right|^2 A(T)$$

where $\alpha(T) > 0$ is the weight for the triangle $T$. $\alpha(T)$'s are all set to be 1 at the first step. Later the weights are adjusted adaptively to penalize the unequal distributions of areal distortion among the mesh triangles.

**Table 1.** Comparison of the three flattening methods.

| Feature/Capability | | FreeSurfer | CirclePack | LSCM |
|---|---|---|---|---|
| Premise for flattening | | miminize metric distortion | preserve angular proportion | preserve angular proportion |
| Is the premise realizable? | | Not guaranteed | possibly | Yes |
| What Information needed? | | Metric and combinatoric | Combinatoric | Metric and combinatoric |
| Mathematically Unique? | | No | Yes | Yes |
| Spherical map | | Yes | Yes | Yes |
| Planar map | Unspecified region | Yes | Yes | Yes |
| | Specified region | No | Yes | Yes |

## Measurements of Distortion

### Angular Distortion

*Angular distortion* is defined to reflect the difference between corresponding angles of the cortical surface $\mathcal{K}$ and its flat map $\mathcal{U}(\mathcal{K})$:

$$\mathcal{F}_{ang}(\mathcal{U}(\mathcal{K})) = \frac{1}{3m}\sum_{Face(\mathcal{K})}^{T_{ijk}} \left(|\theta^{\mathcal{U}}_{ijk} - \theta_{ijk}| + |\theta^{\mathcal{U}}_{jki} - \theta_{jki}| + |\theta^{\mathcal{U}}_{kij} - \theta_{kij}|\right) \tag{3}$$

where $T_{ijk}$ denotes the triangle formed by the vertices $\mathbf{v}_i, \mathbf{v}_j, \mathbf{v}_k$, $\theta_{ijk}$ denote the angle $\angle \mathbf{v}_i\mathbf{v}_j\mathbf{v}_k$ on $\mathcal{K}$ and $\theta^{\mathcal{U}}_{ijk}$ denote the angle $\angle \mathcal{U}(\mathbf{v}_i)\mathcal{U}(\mathbf{v}_j)\mathcal{U}(\mathbf{v}_k)$ on $\mathcal{U}(\mathcal{K})$. All angles on the cortical surface are normalized using the so-called "market share" of angles at vertices.

### Metric Distortion

The first *metric distortion* reflecting the global information (metric distortion–I) is measured as follows:

$$\mathcal{F}_{met-I}(\mathcal{U}(\mathcal{K})) = \min_{s\in\mathbb{R}^+} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{\tilde{N}}\sum_{j\in N(i)} \frac{|s\cdot d^{\mathcal{U}}_{i,j} - d_{i,j}|}{d_{i,j}}\right) \tag{4}$$

where $N(i)$ is the pre-determined index set of neighbor vertices of the vertex $\mathbf{v}_i$ and $\tilde{N} = \mathrm{Card}(N(i))$. Here $s > 0$ is a scaling parameter used with minimization process to avoid the influence of similarity transformations. The second *metric distortion* reflecting the local information (metric distortion–II) is measured as follows:

$$\mathcal{F}_{met-II}(\mathcal{U}(\mathcal{K})) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{\tilde{N}}\min_{s_i\in\mathbb{R}^+}\sum_{j\in N(i)} \frac{|s_i\cdot d^{\mathcal{U}}_{i,j} - d_{i,j}|}{d_{i,j}}\right) \tag{5}$$

Here we move the minimization process inside the first summation, i.e., the minimization process is done independently on its submesh for each vertex $\mathbf{v}_i$. There are $n$ minimization processes in (5).

- It is easy to see that $\mathcal{F}_{met-I}$ and $\mathcal{F}_{met-II}$ will be the same if $s_i = s$ for $i = 1, \cdots, n$, which means that the subregion $\{\mathbf{v}_j\}_{j\in i\cup N(i)}$ associated with each vertex $\mathbf{v}_i$ is uniformly scaled in its flat map.
- We set $N(i)$ to be the index set of all $k$-neighbors [1] of $\mathbf{v}_i$ for $k \leq K$ for some $K > 0$. In our numerical experiments, $K$ is set to be 15.

## Results

- Left cerebral hemispheres were extracted from high-resolution T1-weighted MRI brain volumes obtained from the Montreal Neurological Institute (MNI) and the University of Pennsylvania (UPENN).

**Table 2.** Mesh information for the cerebral cortical surfaces.

| Cortical Surfaces | | Vertices | Triangles | Boundary Vertices | % of the Hemispshere |
|---|---|---|---|---|---|
| Entire Cortex | MNI | 191,724 | 383,444 | 0 | 100% |
| | UPENN | 146,922 | 293,840 | 0 | 100% |
| Frontal Lobe | MNI | 59,319 | 117,944 | 692 | 31.04% |
| | UPENN | 48,311 | 95,776 | 844 | 32.92% |
| Occipital Lobe | MNI | 27,649 | 54,796 | 500 | 14.34% |
| | UPENN | 17,102 | 33,823 | 379 | 11.46% |
| Parietal Lobe | MNI | 37,884 | 74,921 | 845 | 19.74% |
| | UPENN | 28,812 | 56,915 | 707 | 19.66% |
| Temporal Lobe | MNI | 42,631 | 84,568 | 692 | 21.80% |
| | UPENN | 33,971 | 67,365 | 575 | 22.03% |

### Cerebral hemispheres

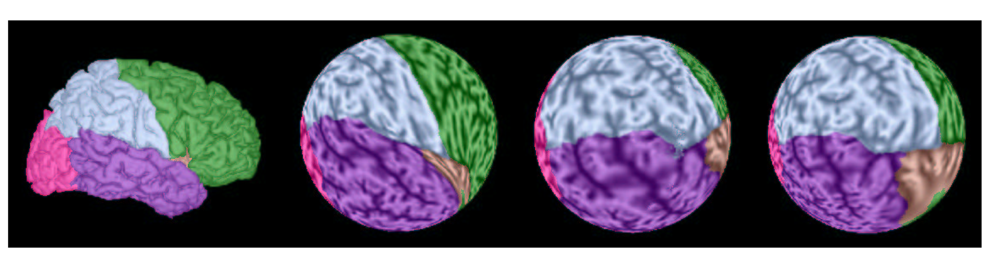

**Figure 1.** From left to right: the MNI left cerebral hemispherical cortex and spherical maps produced by FreeSurfer, CirclePack and LSCM.

**Table 3.** CPU time, angular and metric distortion of spherical maps of left cerebral hemispherical cortices produced by the FreeSurfer, CirclePack and LSCM.

| Cerebral Hemisphere | | | | | |
|---|---|---|---|---|---|
| Spherical Map | | | **FreeSurfer** | **CirclePack** | **LSCM** |
| MNI | CPU Time (min.) | | 630.5 | - | 9.8 |
| | Angular Distortion | Mean Val. | 18.75° | 16.55° | 4.63° |
| | | Std. Dev. | 15.83° | 15.18° | 4.57° |
| | Metric Distortion–I | Mean Val. | 26.06% | 37.86% | 33.70% |
| | | Std. Dev. | 12.37% | 22.48% | 20.10% |
| | Metric Distortion–II | Mean Val. | 18.88% | 20.84% | 11.79% |
| | | Std. Dev. | 7.49% | 13.84% | 7.06% |
| UPENN | CPU Time (min.) | | 384.3 | - | 19.2 |
| | Angular Distortion | Mean Val. | 18.76° | 16.33° | 7.21° |
| | | Std. Dev. | 16.01° | 14.95° | 11.01° |
| | Metric Distortion–I | Mean Val. | 21.57% | 39.81% | 34.33% |
| | | Std. Dev. | 10.02% | 24.36% | 24.39% |
| | Metric Distortion–II | Mean Val. | 16.16% | 18.95% | 14.94% |
| | | Std. Dev. | 7.61% | 13.88% | 11.85% |

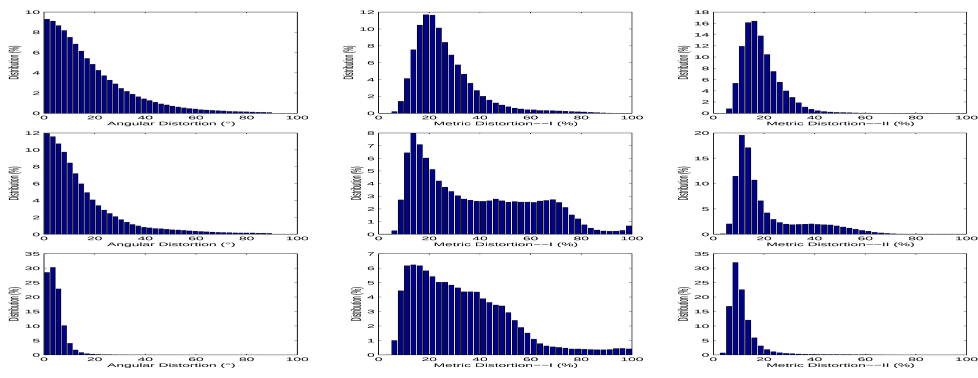

**Figure 2.** Frequency histograms illustrating the angular and metric distortion I and II of spherical maps of the cerebellar cortex generated by FreeSurfer (top row), Circle Pack (middle row) and LSCM (bottom row).

### Lobar patches

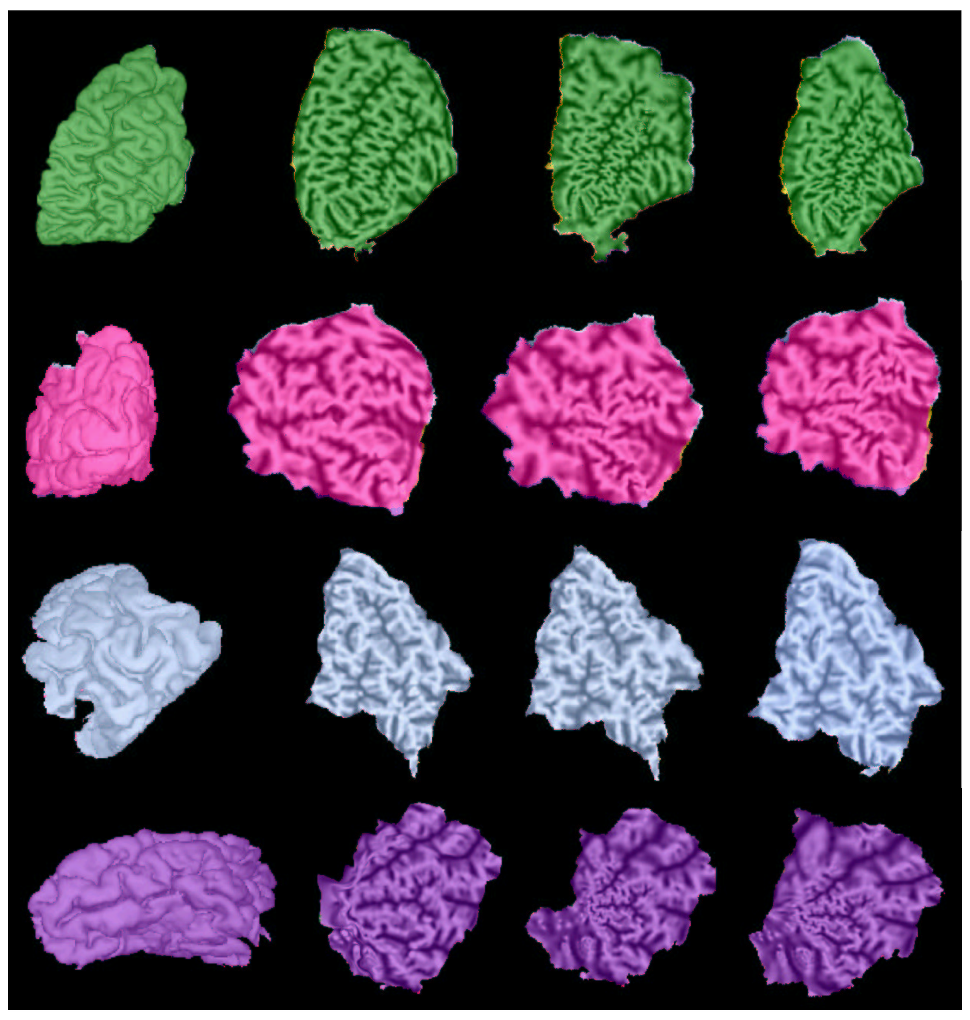

**Figure 3.** From left to right: lobar patches and their planar maps generated by FreeSurfer, CirclePack and LSCM, respectively. From top to bottom (row): frontal, occipital, parietal and temporal patches.

**Table 4.** CPU time, angular and metric distortion of planar maps of the frontal lobar patch produced by the FreeSurfer, CirclePack and LSCM.

| Frontal Lobar Patch | | | | | |
|---|---|---|---|---|---|
| Planar Map | | | **FreeSurfer** | **CirclePack** | **LSCM** |
| MNI | CPU Time (min.) | | 276.5 | - | 56.8 |
| | Angular Distortion | Mean Val. | 11.37° | 11.40° | 4.85° |
| | | Std. Dev. | 10.54° | 10.70° | 6.49° |
| | Metric Distortion–I | Mean Val. | 14.25% | 30.19% | 26.00% |
| | | Std. Dev. | 8.13% | 16.37% | 15.56% |
| | Metric Distortion–II | Mean Val. | 11.16% | 13.09% | 10.57% |
| | | Std. Dev. | 4.75% | 5.78% | 4.72% |
| UPENN | CPU Time (min.) | | 279.4 | - | 61.4 |
| | Angular Distortion | Mean Val. | 15.11° | 12.11° | 5.67° |
| | | Std. Dev. | 15.14° | 11.02° | 8.70° |
| | Metric Distortion–I | Mean Val. | 17.54% | 28.25% | 24.62% |
| | | Std. Dev. | 11.12% | 16.29% | 16.37% |
| | Metric Distortion–II | Mean Val. | 14.15% | 14.89% | 13.31% |
| | | Std. Dev. | 7.70% | 9.85% | 9.16% |

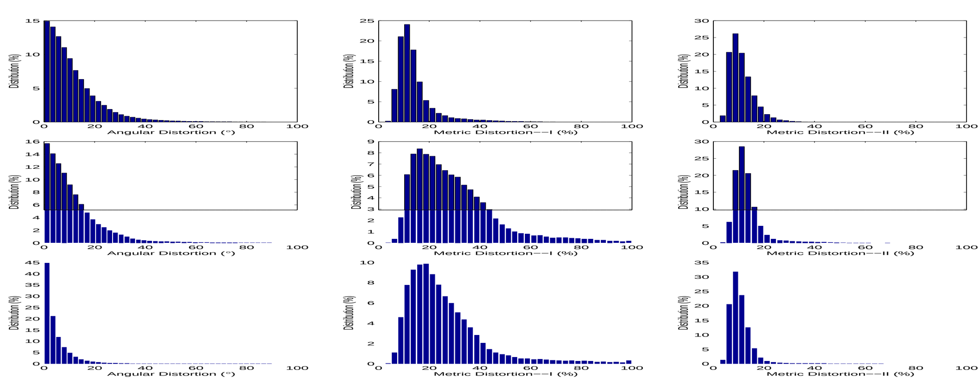


**Figure 4.** Frequency histograms illustrating the angular and metric distortion I and II of planar maps of the MNI frontal lobar patch generated by FreeSurfer (top row), CirclePack (middle row) and LSCM (bottom row).

## Conclusions

- LSCM preserved local angular information during flattening whereas CirclePack did not perform as well as expected due to the fact that the triangles of the cortical meshes were not equilateral.
- For all lobar patches FreeSurfer outperformed both conformal methods with regard to the preservation of metric information-I; however, for the cerebral hemispheres, LSCM performed nearly as well as FreeSurfer and was clearly superior to FreeSurfer and CirclePack with regard to the preservation of angular information, metric information-II and computational efficiency.
- By preserving angular information and adequately preserving metric information, conformal methods such as LSCM may offer advantages for some neuroscience applications over methods such as FreeSurfer, which preserve only metric information and are computationally inefficient.

## References


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