

MAC 2313, Section 05
Test 2

Name: _____

SSN: _____

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks, textbooks and journals are NOT allowed. This test will be graded out of 50.

1. (10 marks) Use a double polar integral to show that the volume of a sphere of radius a is $\frac{4}{3}\pi a^3$.

2. (10 marks) Find the maximum and minimum values of the function $f(x, y) = x^2 - xy + y^2$ subject to the constraint $x^2 + y^2 \leq 8$.

3. (5 marks) Find the quadratic Taylor polynomial approximation to the function $f(x, y) = \sin(2x + y)$ near $(0, 0)$.

4. (10 marks) A rectangular box open at the top is to have a volume of 32 cubic meters. What are the dimensions of the box so that the total surface is a minimum.
Surface area of box = $S = lw + 2lh + 2wh$ where l = length, w = width, h = height.
Volume of box = $V = lhw$.

5. (10 marks) Find the mass of the solid in the first octant bounded below by the xy -plane, laterally by the xz -plane and the plane $y = x$ and above by the cylinder $x^2 + z^2 = 1$, having density $\delta(x, y, z) = z$.

6. (5 marks) Evaluate $\frac{\partial f}{\partial t}$ given $f(x, y) = e^{(x^2+y^2)}$ where $x = r \cos(t)$ and $y = r \sin(t)$.

Bonus (5 marks) Sketch the region over which the following integral is evaluated and evaluate the integral:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \left(z^2 \sqrt{x^2 + y^2 + z^2} \right) dz dy dx.$$