MAC 2313, Section 05 with Dr. Hurdal

Test 3

Name: ________________________ SSN: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. This test will be graded out of 60.

1) (10 marks) (a) Use Green's Theorem to calculate the circulation of $\vec{F} = xy\vec{\imath} + y^2\vec{\jmath}$ around the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$ and oriented counter clockwise.

2) (10 marks) (a) Sketch the vector field $\vec{F}(x, y) = y\vec{\jmath}$.

2(b) Let C_1 be the line segment from $(-1, 1)$ to $(1, 3)$. What is the sign of $\int_{C_1} \vec{F} \cdot d\vec{r}$? Justify your answer.

2(c) Let C_2 be the line segment from $(0, 1)$ to $(2, 1)$. What is the sign of $\int_{C_2} \vec{F} \cdot d\vec{r}$? Justify your answer.

2(d) Let C_3 be the circle with radius 1 and center $(0, 0)$, oriented counter clockwise. What is the sign of $\int_{C_3} \vec{F} \cdot d\vec{r}$? Justify your answer.

2(e) Does this vector field seem to be path-independent? Justify your answer.

3) (10 marks) Find the work done by the force field $\vec{F}(x, y) = (x + z)\vec{\imath} + y\vec{\jmath} - xyz\vec{k}$ in moving a particle along the circle of radius 2 in the plane $z = 2$, centered at (2,2,2) from $(4, 2, 2)$ to $(2, 4, 2)$.

4) (10 marks) Compute the flux of the vector field $\vec{F} = x\vec{\imath} + y\vec{\jmath} + 2z\vec{k}$ through the plane $z = x + 2y$ between $0 \leq x \leq 2$ and $0 \leq y \leq 3$.

5) (10 marks) (a) Use the curl test to demonstrate that the vector field

$$\vec{F}(x,y) = (y^2 + y\sin(xy))\vec{\imath} + (2xy - y + x\sin(xy))\vec{\jmath}$$

is a gradient field.

(b) If C is the curve parameterized by $\vec{r}(t) = (1-t)\vec{\imath} + (10-5t)\vec{\jmath}$ for $1 \le t \le 2$, use the fundamental theorem of line integrals to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

6) (10 marks) A parametric surface S is given by $\vec{r}(s,t) = s\cos(t)\vec{\imath} + s\sin(t)\vec{\jmath} + s\vec{k}$, for $1 \leq s \leq 2$, $0 \leq t \leq 2\pi$.

(a) Find an equation of the form $z = f(x,y)$ that describes this surface and sketch this surface.

(b) If $\vec{F} = x(1+z)\vec{\imath} + y(1+z)\vec{\jmath} + (x^2+z)\vec{k}$, find the flux of $\vec{F}$ across the surface oriented upward.

Bonus (6 marks): Suppose that $\vec{F}(x,y,z) = F_1(x,y,z)\vec{\imath} + F_2(x,y,z)\vec{\jmath} + F_3(x,y,z)\vec{k}$ is a smooth, path-independent vector field. If f, g and h are also smooth functions, show that

$$\vec{G}(x,y,z) = (F_1(x,y,z) + f(x))\vec{\imath} + (F_2(x,y,z) + g(y))\vec{\jmath} + (F_3(x,y,z) + h(z))\vec{k}$$

is also path-independent.