MAC 2313, Section 03
Test 3

Name: ________________________ SSN: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. This test will be graded out of 55.

1. (10 marks) Compute the flux of the vector field $\vec{F} = xz\vec{\imath} + yz\vec{\jmath}$ through the part of the sphere $x^2 + y^2 + z^2 = 9$ oriented outward, with $x \geq 0, y \geq 0, z \geq 0$.

2. (15 marks) Consider the vector fields $\vec{F} = 3y\vec{\imath} + 5x\vec{\jmath}$ and $\vec{G} = 3x\vec{\imath} + 5y\vec{\jmath}$. The curve C_1 is the circle with center $(2, 2)$ and radius 1 oriented counter clockwise and C_2 consists of the straight line segments from $(0, 4)$ to $(0, 1)$ and then to $(3, 1)$. Find the following line integrals and explain all your reasoning.

(a) $\int_{C_1} \vec{F} \cdot d\vec{r}$ (b) $\int_{C_2} \vec{F} \cdot d\vec{r}$ (c) $\int_{C_1} \vec{G} \cdot d\vec{r}$ (d) $\int_{C_2} \vec{G} \cdot d\vec{r}$

3. (5 marks) Let $\vec{F} = 2\vec{\imath} + 5\vec{\jmath}$ be a constant velocity field. Sketch the vector field and find the flow line of $\vec{F}$ that passes through the origin at time $t = 2$.

4. (5 marks) Find the tangent line to the curve $\vec{r} = t^2\vec{\imath} + 2t^3\vec{\jmath} - 2t\vec{k}$ at the point $(1, 2, -2)$.

5. (10 marks) Compute the flux of the vector field $\vec{F} = y\vec{\imath} - x\vec{\jmath} + z\vec{k}$ through the parametric surface $x = s+t, y = s-t, z = s^2+t^2$ oriented away from the origin, where $0 \le s \le 1, 0 \le t \le 1$.

6. (10 marks) Sketch the vector field $\vec{F} = -x\vec{\jmath}$. Arrange the line integrals $\int_{C_1} \vec{F} \cdot d\vec{r}, \int_{C_2} \vec{F} \cdot d\vec{r}, \int_{C_3} \vec{F} \cdot d\vec{r}$ in ascending order where C_1 is the straight line from $(-1,-1)$ to $(1,1)$, C_2 is the arc of a circle of radius 1 centered at the origin, from $(1,0)$ to $(0,1)$ and C_3 is the line segment from $(-1,-1)$ to $(-1,0)$ and then to $(-2,0)$. Do you think $\vec{F}$ is a gradient vector field? Justify all your answers.

Bonus (4 marks): Use Green's Theorem to show that the line integral of $\vec{F} = (y^4 - y)\vec{\imath} + 4y^3x\vec{\jmath}$ around a simple closed curve in the xy-plane, oriented as in Green's Theorem, measures the area enclosed by the curve.

Bonus (2 marks): Identify the parametric surface from question 5.

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