

1. Let R be the relation on the set \mathbb{Z} of all integers, where aRb means that $ab \leq 0$. Is R an equivalence relation?

2. Let \mathbb{R}^2 be the set of all ordered pairs of real numbers; that is, $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$. Define a relation R on \mathbb{R}^2 , where $(x_1, y_1)R(x_2, y_2)$ means that $x_1 + y_2 = x_2 + y_1$.

(a) Prove that R is an equivalence relation.

(b) Find the equivalence class $[(1, 2)]$.

3. Let R be the relation on the set \mathbb{Z} of all integers, where aRb means that $a^2 = b^2$. Determine whether R is reflexive, symmetric, antisymmetric, transitive.

4. Find the matrix that represents the given relations, where the elements are ordered as indicated.

(a) R on $\{1, 2, 3, 4, 6, 12\}$, where aRb means $a|b$.

(b) R^2 , where R is the relation on the set $\{w, x, y, z\}$ given by

$$R = \{(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, y), (z, z)\}.$$

5. Determine whether the relation on the set $\{a, b, c, d\}$ defined by the matrix M_R below (elements are ordered as indicated) is reflexive, symmetric, antisymmetric, transitive.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

6. Draw the Hasse diagram of the relation R on the set $A = \{2, 3, 4, 6, 10, 12, 16\}$, where $(a, b) \in R$ means that $a|b$.

7. Find the matrix of the transitive closure of the relation R on the set $\{a_1, a_2, a_3\}$ whose matrix in the given order is

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$