1. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$ and use it to solve the equation $Ax = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 1 & 4 & -2 & 3 & -2 \\ -2 & -8 & 4 & -5 & 7 \\ 3 & 12 & -6 & 9 & -9 \end{bmatrix}$.

(a) Find the rank of A.
(b) Find a basis of the null space of A.
(c) Find a basis of the column space of A.

3. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 4 \end{bmatrix}$.

a. Find the eigenvalues of A; for each eigenvalue λ, find a basis of the corresponding eigenspace of A.

b. Is A diagonalizable? Why?

4. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

5. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 0 \\ 2 & -1 & 2 \end{bmatrix}$.

6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x) = Ax$, where $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. Find the matrix $[T]_{\mathcal{B}}$ of T relative to the basis $\mathcal{B} = \{b_1, b_2\} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$.

7. A 2×2 matrix has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 1/2$ with corresponding eigenvectors $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Find a formula for A^k, where k is a positive integer.

8. True or False?

(a) If A is an $n \times n$ matrix such that the linear transformation $T(x) = Ax$ is one-to-one, then A is invertible.

(b) The null space of a 4×5 matrix of rank 4 must be trvial; that is, it must equal $\{0\}$.

(c) If an $n \times n$ matrix A has n linearly independent eigenvectors, then A has n distinct eigenvalues.

(d) If $\lambda = 0$ is an eigenvalue of A, then A is not invertible.