1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$.

(a) Find the characteristic polynomial and the eigenvalues of A.
(b) For each eigenvalue of A, find a basis for the corresponding eigenspace.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \\ -1 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$.

(a) Find a basis for the orthogonal complement, $(\text{Col}\, A)^\perp$, of the column space of A.
(b) Find an orthogonal basis for $(\text{Col}\, A)^\perp$.

3. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors u_1, u_2, u_3, where $u_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$ and $u_3 = \begin{bmatrix} 1 \\ -4 \\ 4 \\ -1 \end{bmatrix}$.

4. Let $Q = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \\ 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$, $R = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$. If $A = QR$ is the QR-factorization of A, find the least-squares solution of $Ax = b$ and the orthogonal projection $\hat{b}$ of the vector b onto $\text{Col}\,(A)$, without computing the matrix A explicitly.

5. Consider the data set $D = \{(-2,1), (0,2), (1,3), (2,3)\} \subset \mathbb{R}^2$.

(a) Find the equation of the least-squares line $y = \beta_0 + \beta_1 x$ for the data.
(b) Describe the model that produces a least-squares fit by a function of the form $y = \beta_1 x + \beta_3 x^3$. Find the design matrix X, the parameter vector β, and the observation vector y explicitly. You do not need to find β_1 and β_3.

6. Let $u_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -3 \end{bmatrix}$, and $y = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. (Note that $\{u_1, u_2\}$ is an orthogonal set.)

(a) Find the vector in $\text{Span}\,\{u_1, u_2\}$ that is closest to y.
(b) Find the distance from y to $\text{Span}\,\{u_1, u_2\}$.

7. True or False?

(a) The orthogonal projection of a vector $y \in \mathbb{R}^n$ onto a subspace W gives the vector in W closest to y.

(b) Let U be an $n \times n$ matrix. If the columns of U form an orthonormal set, so do the rows of U.

(c) If the columns of an $n \times n$ matrix are linearly independent, then $A^T A$ is invertible.

(d) The set of all vectors in $\mathbb{R}^3$ that are orthogonal to a fixed nonzero vector $u \in \mathbb{R}^3$ is a 2-dimensional subspace of $\mathbb{R}^3$.