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Research Statement

My current research projects address problems arising in fluid dynamics, especially in geophysical settings and related to hydrodynamic stability. To understand the complex phenomena exhibited in these fluid systems, I use tools and techniques from an array of mathematical subdisciplines with an emphasis on computational methods. My work involves creating models with PDEs to describe fluid motion, numerically simulating these systems to visualize their behavior, and performing analyses to validate results and guide our understanding of the theory. Individually, each of these steps is non-trivial; modeling with PDE systems, developing and producing *useful* numerical simulations, and conducting analyses all require expertise. However, studying fluids and understanding how they behave have been questions of interest for over two thousand years– Archimedes and his eponymous principle concerning fluid buoyancy from around 250 BCE are some of the earliest pieces of evidence for this.

The problems I work on have genuine physical applications and are closely tied to areas of study outside of mathematics. My first project below details how groundwater and surface-water interact in settings where convection plays a role. The main findings of this investigation are several analytical results on stability I rigorously proved in [5, 6] along with numerical simulations. The second project is a collaborative effort with a fellow Florida State alumnus concerning how sinkholes form, expand, and then collapse. Tools used in this endeavor deal with modeling the physics of this system and then using computational methods to visualize our results. The third project listed in this statement is related to studying and improving the removal of crude oil by-products in 'tailings ponds', ponds constructed by oil companies to capture residual material from the oil extraction process. In Canada, these tailings ponds cover around 85 sq. miles– as reported in 2017– and pose major environmental issues to both humans and wildlife. With a (potential!) grant from Canada's Oil Sands Innovation Alliance, several collaborators and I are actively working toward developing a comprehensive understanding of the tailings-removal process in hopes of accelerating remediation efforts.

I discuss these three projects below in more depth including a description of past, current, and future work. Additionally, I highlight several undergraduate-accessible projects related to my work, including the work of my two undergraduate students and their recently submitted paper, [7].

Convection in coupled fluid-porous media systems

The phenomenon of fluid flowing over a porous medium has been studied for more than a century in a variety of settings. Chief among these are geophysical applications, such as the mixing of surface-water and groundwater, contaminant transport and bioremediation efforts, and flow within oil reservoirs. Given the increasingly urgent need to understand water resources more fully, investigating the interaction between surface-water and groundwater is particularly important.

To begin studying this coupled fluid-porous medium system, I started with numerical simulations. Slightly altering certain parameters in these simulations sometimes led to drastically different flow profiles. By changing only the depth ratio of the two regions, resulting profiles could go from having convection cells occupy the entire domain to having the cells only appear in the fluid region. Adjusting the depth ratio yet again could cause the convection cells to vanish altogether. What I had stumbled upon were different parameter sets for various stability regimes.

With this coupled system, the different stability regimes are analogous to heating up a pot of water

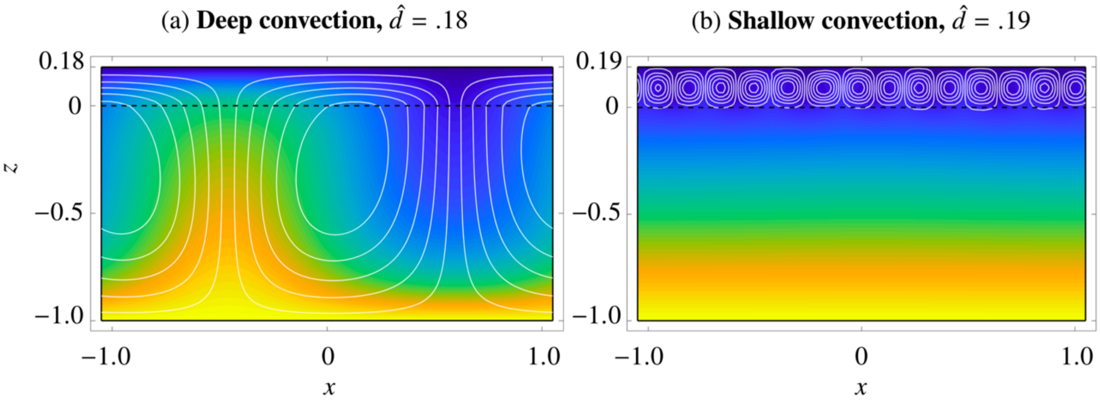


Figure 1: Marginally stable flow configurations and temperature profiles (color) for two values of the depth ratio, $\hat{d}$. In (a), we see a depth ratio of $\hat{d} = .18$ produces convection cells that extend throughout the entire domain, while (b) $\hat{d} = .19$ produces cells that are confined to the free-flow region.

on the stove. If the temperature under the pot is hot enough (over the boiling point), convection cells begin to form in the water before eventually boiling. Conversely, if the temperature under a pot on the stove is not hot enough, then the water will not boil- it remains still. Similarly, if there is a great enough temperature difference between the the top of a lake and the bottom of a sandy or muddy region under the lake, then convection cells will form in the lake. Likewise, if there is not enough of a temperature difference, then water in the lake will remain stagnant.

Determining the stability criteria in fluid-porous media systems had been researched with a variety of models for the equations governing fluid motion [2, 3, 4]. However, no work had been conducted on the Navier-Stokes-Darcy system, the most well-accepted model for fluid-porous systems in geophysical applications. My work in *SIAM Applied Math* [5] provided the most general stability results for this system. The analytical results from this paper help validate my numerical simulations, and narrow parameter regimes of interest. While many projects focus on either analysis or numerical methods, the combination of both is necessary to develop a holistic understanding of how this system behaves. Using analysis in tandem with computational methods is a distinctive feature of my work.

Two standard convection cell profiles from my work are shown in Figure 1. The black lines denote the streamlines of the fluid while the temperature field is shown in color. The first case shows convection cells occupying the entire domain while the second case shows cells confined to the fluid region only. The only difference between these two cases is the depth ratio of the two regions. Altering the depth ratio slightly (from .18 to .19) causes a shift in the convection cells. This observation spurred an additional project: developing a theory to determine and predict parameter regimes necessary for this phase transition in convection to occur [6].

The theoretical results from this project have laid the foundation for a variety of other explorations (both theoretical and numerical) into the Navier-Stokes-Darcy system would make excellent undergraduate research projects. One example would be an investigation into the dynamic reorganization of the flow as convection cells shift from occupying the entire medium to the fluid region alone. This could be two projects, focusing on numerical or theoretical aspects of the system depending on the interests of the student involved. A second project would involve a student developing a decoupled scheme to efficiently simulate this system based on an asymptotic argument, like the argument provided in the appendix of [5].

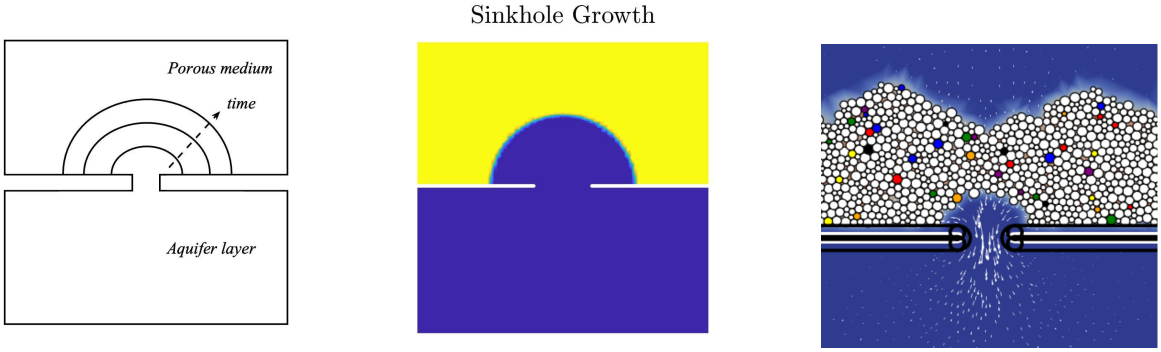
Inception and evolution of sinkholes in karst geometries

Based on experimental results from the Geophysical Fluid Dynamics Institute at FSU, collaborators and I have developed numerical methods and a mathematical framework for detailing how sinkholes form and evolve in time. Since there are few mathematical papers concerning sinkholes and even fewer with numerics, we have developed two independent numerical methods to describe this phenomenon, which we compare against each other to show consistency of our results. Both methods exhibit similar behaviors (qualitative and quantitative), summarized with the simplified schematics shown in Figure 2.

Our first model uses a novel discrete-continuum hybrid method [1]. With this method, we solve for the fluid velocity in the medium and the aquifer layer for a given porosity field. We then update the location of the discrete fluid particles in the medium (and any in the aquifer layer). With the updated particles' locations, we use a fast Gaussian transform to define the porosity field as a continuum which is then used to solve for the fluid velocity, and the process repeats. One unique feature of this method is non-homogeneous porosity field which develops. With this model, we can clearly see the 'weakening' of the medium along the interface as the fluid begins to erode it away. To confirm and verify the results of this model, we developed a second model using a level-set method.

Level-set methods use a function to differentiate between regions of a domain in a manner similar to elevation contours/isolines on a map. For our work with sinkholes, if a region has isolines above a certain value, we are in the porous medium. Alternatively, when the level-set function is less than a certain value, we are in the aquifer layer. Since our variables are defined over the entire domain, the level-set function allows us to distinguish between values of these variables in the two regions. The hardest part of this method is determining the equations to describe how the level-set function progresses, i.e., detailing how the interface between the medium and aquifer layer erodes and evolves in time.

The level set method can be approachable for an undergraduate student with some background in coding, as we could discuss the mathematical framework for the method with an introduction to partial differential equations. This work would ideally be done by a student with a desire to attend graduate school in applied or computational math, or by a student who wants to see if mathematical research would be a good fit for them.



(a) Schematic for the evolution of a sinkhole.

(b) Simulations with the level-set method.

(c) Simulations with the hybrid discrete-continuum method.

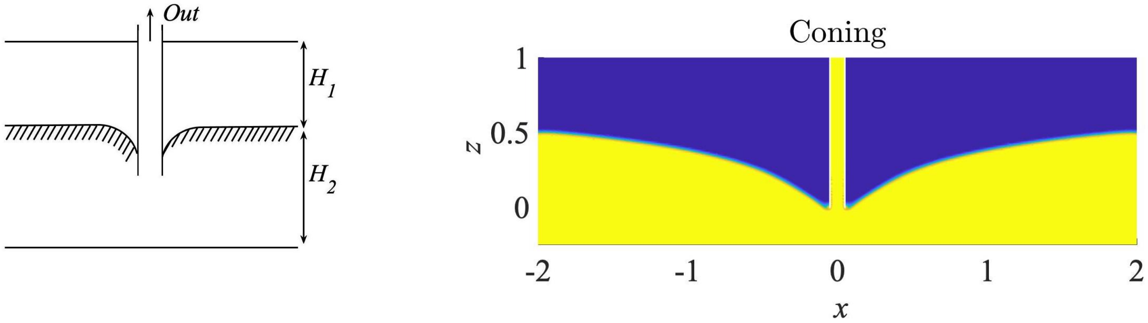
Figure 2: The interface between the medium and the fluid layer evolves in time, creating a cavity in the porous medium. As this cavity expands, the sinkhole is created. Once the cavity becomes too large to support the solid layer above it, the sinkhole collapses.

Coning during extraction from tailing ponds

In this project with collaborators in engineering, we (will hopefully soon!) have a grant sponsored by Canada's Oil Sands Innovation Alliance to study the phenomenon of coning. The Canadian oil-sand (a form of crude oil) industry has grown rapidly in the past several decades due to the development of newer, more profitable technology for oil production. As a result, these operations have produced considerable amounts of excess material discharged into ponds. These excess discharges, called 'tailings,' contain clay, silt, and crude-oil petroleum suspended in water. After years of particle settling, water forms at the top of these ponds and the clay-like tailings suspension forms near the bottom. Removing tailings from these ponds has a significant impact on the environment, as well as companies' operational costs. To remedy environmental effects and reduce tailing-extraction costs, companies like Canada's Suncor and Syncrude have begun developing more advanced tailing-removal processes. During the extraction process, companies only want to remove the tailings and not the water which has settled atop. When removing the tailings (a non-Newtonian fluid) via a pipe, a cone-shaped feature begins to form along the pipe and the water over the tailings begins to be extracted as well. This is shown in Figure 3, which included a schematic as well as results from my numerical simulations.

The outcome of this research is a comprehensive understanding of the parameters involved in the coning behavior during extraction of tailings. Results from this project will also be shared with engineers to aid them in rationally designing tailings-extraction strategies in order to minimize water intrusion (from the top layer). This will potentially save the oil-sand industries millions of dollars in operational costs and accelerate the reclamation process. To simulate the occurrence of coning during the tailings-extraction process, we use a level-set method, like the one used above in the sinkhole project. The next step of this project concerns determining how factors (depth ratio of the fluids, pipe diameter, extraction rate, etc.) influence the behavior of coning and impact the effectiveness of removing tailings.

With an introductory course to PDEs (or a crash course), I am confident the level-set method used for this project is simple enough for an undergraduate student to understand, use, and explore in a summer research project or directed independent study. Additionally, with results based on this project, undergraduate students would have the opportunity to present their results at conference talks and possibly in publications.



(a) Schematic for coning.

(b) Simulations of coning with fluid being pumped out of a pipe in the middle of the domain.

Figure 3: Comparison of the benchmark and simulations of the coning phenomenon with my level-set method. This is at the time-step immediately before the top fluid begins being extracted through the pipe along with the the bottom fluid, i.e., the tailings.

Conclusion

Throughout my academic career thus far, I have shown I am an effective, self-motivated researcher, working on projects outside of direct guidance from my advisors. I have sought out opportunities for collaborations with interdisciplinary projects, and will continue to do so as I implement my own research program.

An integral part of my research program includes undergraduate-accessible projects. When I was an undergraduate student, I was exposed to math research through a summer project with one of my professors. This experience was incredibly formative, solidifying my desire to attend graduate school. As a result, I want to pass this on to students, helping mentor them as well as introducing them to mathematical research along the way. So far, I have mentored two female undergraduate students at Trinity College with a summer research project which culminated in submitting an article for publication and presentations at a research symposium.

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