1. Section 5.4 Solving Polynomial and Rational Inequalitites

The Sign Chart Method (usually taught in College Algebra)

Step 1. Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \qquad f(x) \geq 0$$
$$f(x) < 0 \qquad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient, and find the domain of f.

Step 2. Determine the real numbers at which the expression f equals zero and, if the expression is rational, the real numbers at which the expression f is undefined.

Step 3. Use the numbers found in Step 2 to separate the real number line into intervals. These numbers are called **partition points**.

Step 4. Select a number in each interval and evaluate f at the number.

(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.

(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.

If the inequality is not strict ($\leq$ or $\geq$), include the solutions of $f(x) = 0$ that are in the domain of f in the solution set. Be careful to exclude values of x where f is undefined.

Remark 1.1. *This method works because a polynomial or rational function can only change signs where it equals zero or is undefined.*

Remark 1.2. *If a partition point comes from a linear factor with odd multiplicity, $f(x)$ will change sign on either side of the partition point. If the linear factor has even multiplicity, the sign of $f(x)$ will not change.*

Example 1.1. *Solve* $\dfrac{1}{3-x} \geq 2$.

Example 1.2. *Solve* $\dfrac{4x-1}{(x+2)(x+3)} \leq 0$.

(A) $[-3,-2] \cup [1/4,\infty)$

(B) $(-3,-2) \cup [1/4,\infty)$

(C) $(-\infty,-3] \cup [-2,1/4]$

(D) $(-\infty,-3) \cup (-2,1/4]$

Remark 1.3. *The union symbol in sets is not the same as the letter U and does not appear on your keyboard. To enter the union in MML you have to use the math tool palette.*

2. Solve Polynomial and Rational Inequalities Graphically

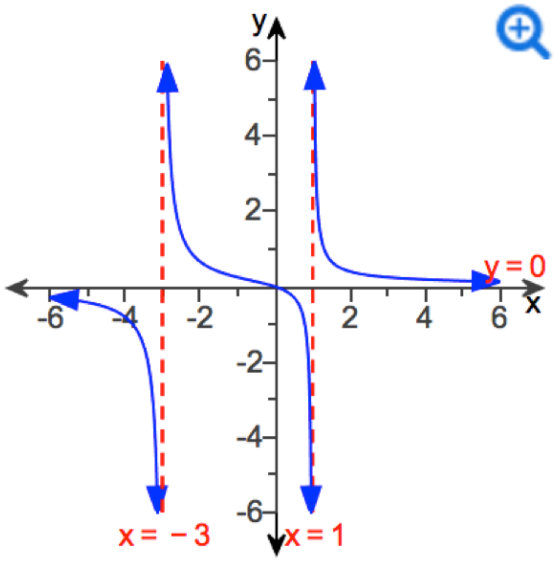
Example 2.1. *Solve the inequality* $f(x) \geq 0$*, where* $f(x) = 2(x+3)(x-2)^3$

Remark 2.1. *If a partition point comes from a linear factor with odd multiplicity,* $f(x)$ *will change sign on either side of the partition point. If the linear factor has even multiplicity, the sign of* $f(x)$ *will not change.*

Example 2.2. *Use the graph of the function f to solve the inequality.*

(a) $f(x) > 0$

(b) $f(x) \leq 0$



We can use a combination of the sign chart and information about the graphs of functions that we have learned this semester to make the solution quicker to find.

Example 2.3. *Solve* $-2(x+4)^2(x^2+9)(4-x)^3 > 0$.

Example 2.4. *Solve* $\dfrac{x^4(x^2+4)(x^2-4)}{(2x+1)^2(x-4)^3} \leq 0$.