Two-dimensional Rayleigh-Bénard convection with an obstruction

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Objective

We use the system of partial differential equations (PDEs) composed of the Navier-Stokes-Boussinesq equation and an advection-diffusion equation for heat to investigate how an obstruction affects the natural convection of an incompressible fluid inside a square enclosure heated from below.

Introduction

Rayleigh-Bénard convection describes natural convection where a fluid is heated from below. When a fluid is heated, it becomes less dense; therefore, hotter fluid rises due to buoyancy, while cooler fluid sinks. This problem has been the focus of much published literature due to its many applications in engineering, physics, and mathematics. However, literature detailing this kind of convection *with an obstruction* in the domain is much less frequent.

To describe how the fluid behaves as a result of a temperature difference within an enclosure, we consider the following PDEs: the Navier-Stokes-Boussinesq equation, the incompressibility condition, and the advection-diffusion equation, which are respectively:

$$\begin{cases} \frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla p + (\frac{Pr}{Ra})^{\frac{1}{2}}\nabla^2\vec{u} + \theta\vec{k}, \\ \nabla\cdot\vec{u} = 0, \\ \frac{\partial\theta}{\partial t} + \vec{u}\cdot\nabla\theta = \frac{1}{(RaPr)^{\frac{1}{2}}}\nabla^2\theta, \end{cases}$$

where the velocity vector $\vec{u}$, pressure p, and temperature θ are variables of interest, $\vec{k}$ is the upward pointing unit normal vector, while Pr and Ra are the Prandtl and the Rayleigh numbers, respectively. The Rayleigh number is a non-dimensional constant describing the heat difference between the top and bottom surfaces of the enclosure, and the Prandtl number, Pr, is a dimensionless ratio of momentum diffusivity to thermal diffusivity.

The velocity fields and temperature profiles in Figure 1 are computed for an enclosure without an obstruction, allowing us to use the work of Ouertatani et al. (2008) as benchmark for our numerical solutions.

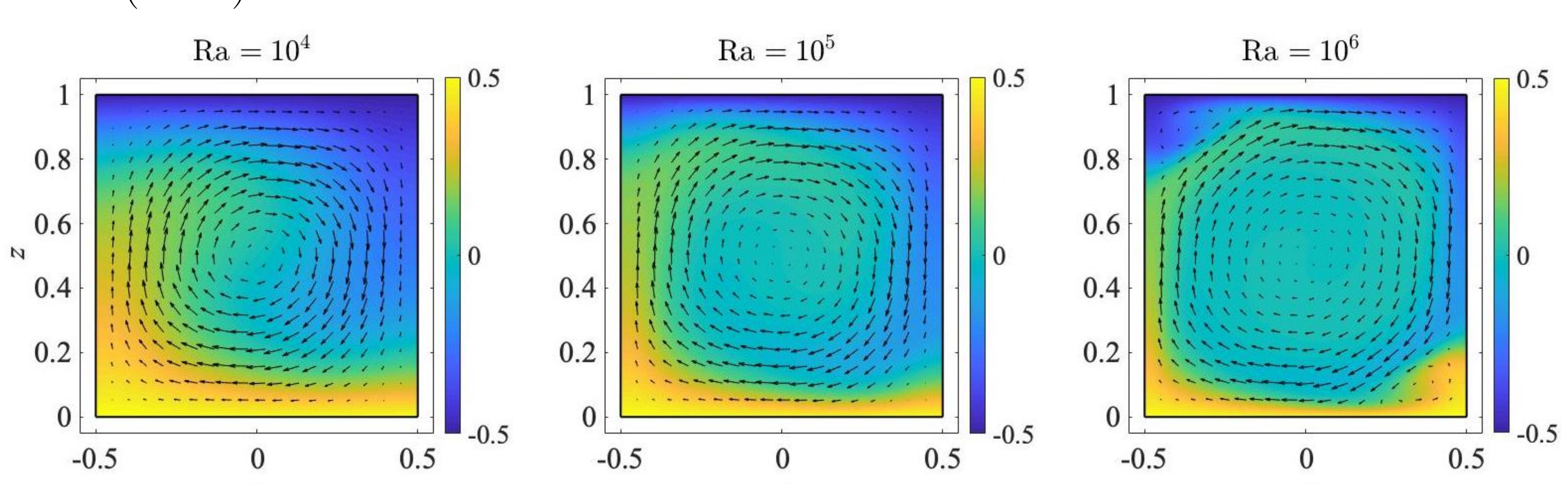

Figure 1: Temperature and velocities fields for cases without an obstruction.

Numerical simulations

Since exact analytical solutions are, in general, impossible to find, we use a finite element method (FEM) to approximate solutions to the system of PDEs. For our mesh, the unit square domain is discretized and divided into a finite number of small triangular elements, shown in the figure below. Then, the weak form of our system is evaluated and satisfied on each element of the domain.

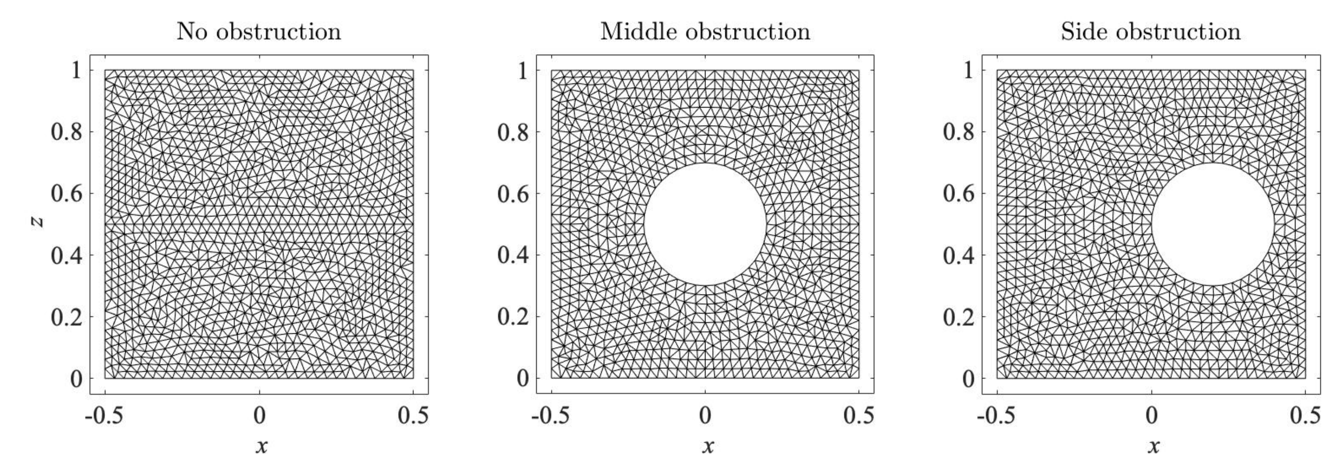

Figure 2: Examples of our discretized meshes.

One of our main quantitative benchmarks is the local Nusselt number, defined below, that quantifies the vertical flux due to convection. More vigorous convection correlates to a higher Nu_L value at the boundary.

$$Nu_L(x) = -\frac{\partial\theta}{\partial z}\bigg|_{z=0}.$$

The cases without an obstruction were considered in Ouertatani et al. (2008), and our results show good agreement with theirs; this comparison helped confirm that our code was working as it should. We then added in an obstruction to see if it influenced the local Nusselt numbers– these results are shown in Figure 3. For the middle obstruction results in the left panel, we see the $Nu_L(x)$ profiles follow similar paths; this suggests that an obstruction in the middle of the domain does not notably influence convection. However, for the obstruction towards the side of the domain (right panel), we obtained qualitatively different behavior compared to the cases without an obstruction, suggesting that the location of obstructions can significantly alter convection.

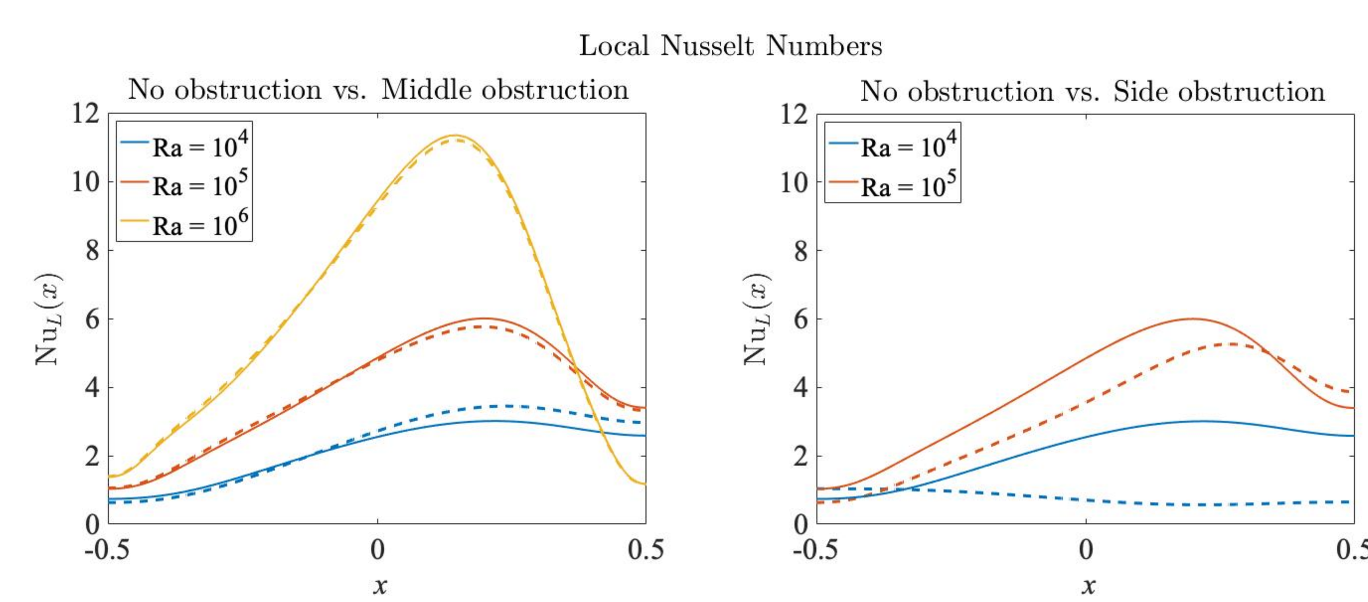

Figure 3: Local Nusselt numbers on the lower boundary of the domain. On the left: no obstruction (solid) vs. middle obstruction (dashed), and on the right: no obstruction (solid) vs. side obstruction (dashed).

Results

The cases with the middle obstruction all achieved a steady-state, similar to the cases without an obstruction. However, the side obstruction cases did not always settle down. For example, with $Ra = 10^6$ and the side obstruction, we observed periodic behavior– a stark contrast to cases with no obstruction and the middle obstruction– suggesting that the location of an obstruction can produce noticeably different behavior with convection.

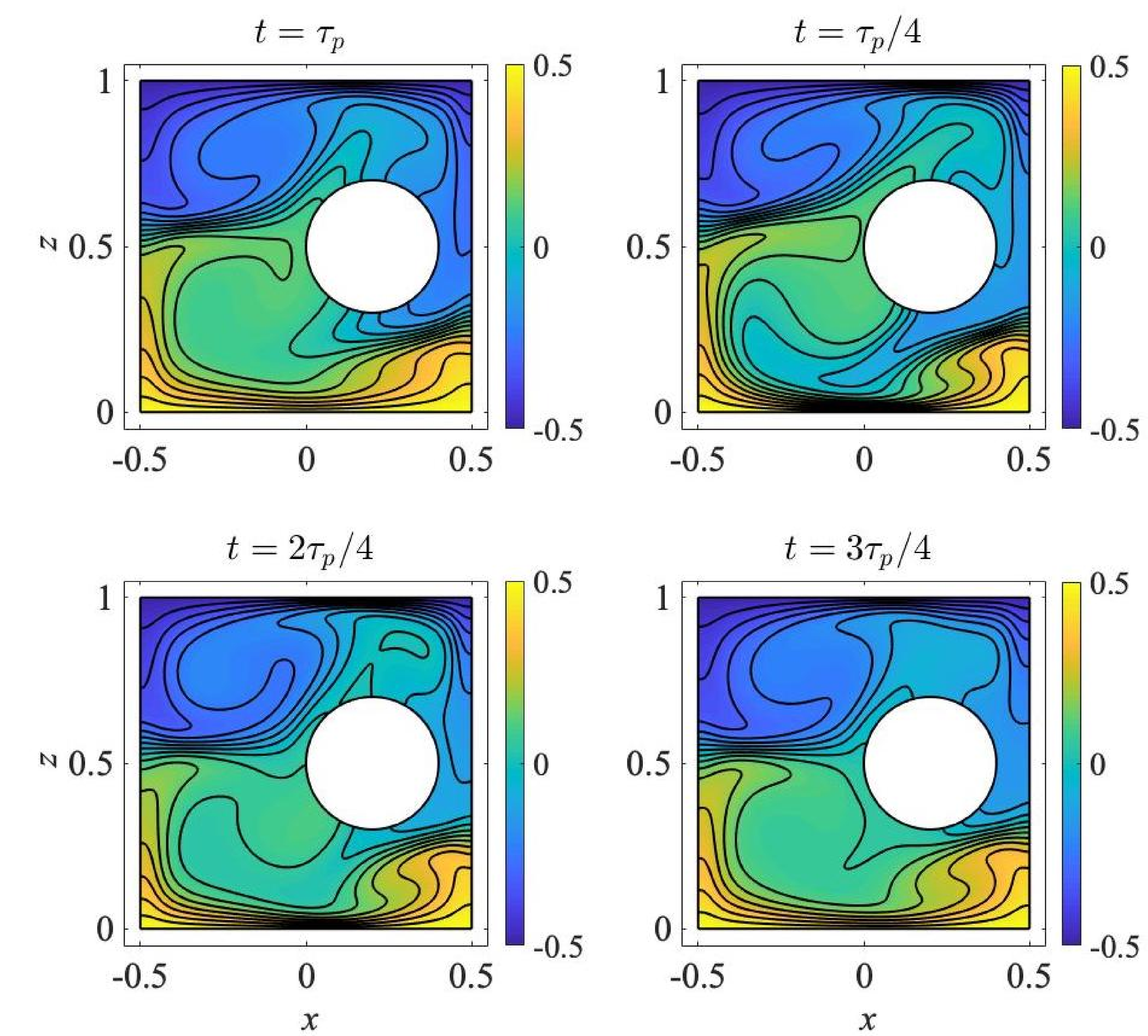

Figure 4: Temperature profiles for $Ra = 10^6$ from four times in one period.

Conclusions, and Future Work

The centered circular obstruction we investigated had no significant effect on convection. However, obstruction towards the side can affect significantly convection. Future work will investigate how the size and placement of obstructions affects convection.

References

N. Ouertatani, N. B. Cheikh, B. B. Beya, and T. Lili. Numerical simulation of two-dimensional rayleigh–bénard convection in an enclosure. *Comptes Rendus Mécanique*, 336(5):464–470, 2008.

Acknowledgements

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