

# Research Statement

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## 1 Introduction

As a modern applied mathematician, my research incorporates a range of subdisciplines, including modeling, asymptotic analysis, numerical computation, applied PDEs, applied complex analysis, dynamical systems, and stochastic analysis. I feel comfortable drawing ideas and techniques from each of these areas to tackle problems that stem from scientific questions. Specifically, most of my recent work focuses on modeling fluid-structure interactions that arise in biology, geophysics, or other applications. By their nature, such systems tend to be highly dynamic and nonlinear. These challenges necessitate a so-called hybrid approach, in which a sound theoretical foundation is combined with advantageous asymptotic reductions and cutting-edge computational tools to address open questions or make new predictions. Problems like these are not only of interest to applied mathematicians, but also to physicists, biologists, environmental scientists, and engineers, which often leads to fruitful collaborations. Below I describe several examples to illustrate the symbiotic relationship between applied mathematics and science, with a few examples arising from biology and a few from geophysics.

## 2 Biocomotion in fluids

For flying and swimming animals alike, wing-or-fin flexibility has long been recognized to improve propulsive performance. Quantifying this phenomenon, though, requires understanding the subtle way that elastic structures couple to the surrounding flows. Recently, I developed a theoretical framework to describe this interaction, based on asymptotic analysis of the Euler equations in the regime of small-flapping amplitude. Because the asymptotics simplify the structure of the PDE-system tremendously, this framework permits *exact solutions* for the emergent wing motions in certain cases. As published in the *JFM* [16], these solutions enable precise characterization of propulsive resonances that have been observed in previous numerical studies, and help explain a surprising retrograde phenomenon observed in laboratory experiments [27].

More recently, I extended these ideas to design a numerical method for handling wings of arbitrarily varying material properties. The method makes use of the small-amplitude fluid flow solutions,

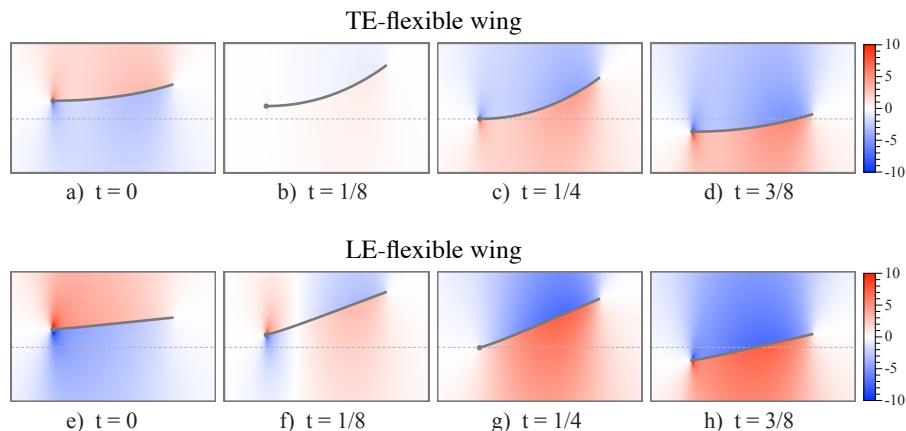


Figure 1: Computed wing deformations for a wing with flexibility concentrated at its trailing edge (top row) and at its leading edge (bottom row), along with the surrounding pressure field.

while describing wing deformations through variable-coefficient Euler-Bernoulli beam theory. The two are linked by the *hydrodynamic load*, which acts to deform the wing and, at the same time, is influenced by the deformations. The important insight is that the load can be thought of as a *nonlocal* operator that acts on the 1D wing kinematics, thereby reducing the dimensionality of the problem. By leveraging the small-amplitude asymptotics, it is possible to compute this operator with near-optimal efficiency,  $O(N \log N)$  operations. A preconditioned iterative method then permits computation of the emergent wing kinematics. Figure 1 shows the flapping motions of a wing with flexibility concentrated at its trailing edge (top row) versus at its leading edge (bottom row), illustrating the different deformations that can arise. A complete description of the numerical method is published in the *Journal of Computational Physics*, where I also analyze singularities present in the nonlocal operator to derive rigorous estimates for the method's accuracy [18].

By virtue of its efficiency, this PDE solver can be inserted into an optimization loop to determine the material distributions that deliver peak performance. As published in *Physics of Fluids*, I considered a flapping wing with variable elastic modulus and asked which distribution maximizes thrust production [17]. I discovered that concentrating flexibility near the driving point can boost thrust substantially. Placing a torsional spring at the root of a rigid wing focuses all of the flexibility at a single point, and this configuration globally optimizes thrust [17]. Intriguingly, this arrangement is observed in the architecture of insect wings, in which the elastic wing-body joint acts as a torsional spring positioned near the wing's leading edge, and the majority of elastic deformations occur along that axis [28]. Very recently, another group followed my procedure to determine material distributions that optimize propulsion in the Stokes regime [22].

Many open questions remain in this line of research. To begin, I envision extending the small-amplitude theory to 3D by leveraging ideas from lifting-line theory, in which 2D cross-sections communicate through a circulation relation. Unlike classical lifting-line theory which is quasi-steady, the extension of the small-amplitude framework would allow one to characterize a range of dynamical states in which the wing deflections interact strongly with the surrounding flows. Another extension would be to adapt the small-amplitude framework to handle multiple interacting propulsors—a quest that has already excited some recent interest [12, 24].

### 3 Fluids shaping boundaries

Effects of flow-driven erosion and dissolution are visible throughout the natural world, ranging from massive landscapes down to small-scale patterns. In these settings, solid boundaries are not only shaped by the flow of air or water, but also influence the surrounding flows. The feedback between the two eventually creates the fascinating morphologies that we observe. Since naturally-occurring examples are dominated by complexity, there is great value to viewing such processes through the lens of idealized problems—much like the classical Stefan problems of simple geometries melting or solidifying. With this mindset, my collaborators and I designed laboratory experiments to examine the processes of erosion and dissolution in highly controlled settings. By combining experimental observations with mathematical models, we were able to extract key principles that underly these shape-changing processes. We discovered a class of equilibrium structures preserved by the shape-flow interaction and a power-law description for how the structures vanish self-similarly in time. Interestingly, we found the emergent morphology to be process dependent, as dissolution forms smooth, rounded surfaces [10], while erosion carves angular features [25, 20]. These studies were published in the *PNAS* [25], *Physics of Fluids* [20], and *JFM* [10]. More recently, Jinzi Mac Huang and myself solved the outstanding problem of the shape formed by a dissolving boding under the action of a self-induced natural convective flow, as published in *PRL*. This shape explains the origin of so-called karst pinnacles found in stone forests around the globe [13].

In a more recent *CPAM* paper, I discovered that the processes of erosion, dissolution, and melting can all be unified under a common theoretical framework [19]. In this paper, I merge separated-flow theory and boundary-layer theory to obtain a class of singular Riemann-Hilbert problems for the equilibrium morphologies that are formed. Because the singularity dictates the opening angle of the body, a class of exact solutions can capture both the rounded surfaces formed by dissolution/melting (an opening angle of  $180^\circ$ ) and the sharp features formed by erosion (a  $90^\circ$  angle). The different morphologies are depicted in the left panel of Fig. 2. The *exactly solvable* model laid out in [19] offers a platform that could be used to design and test numerical methods for simulating naturally-occurring

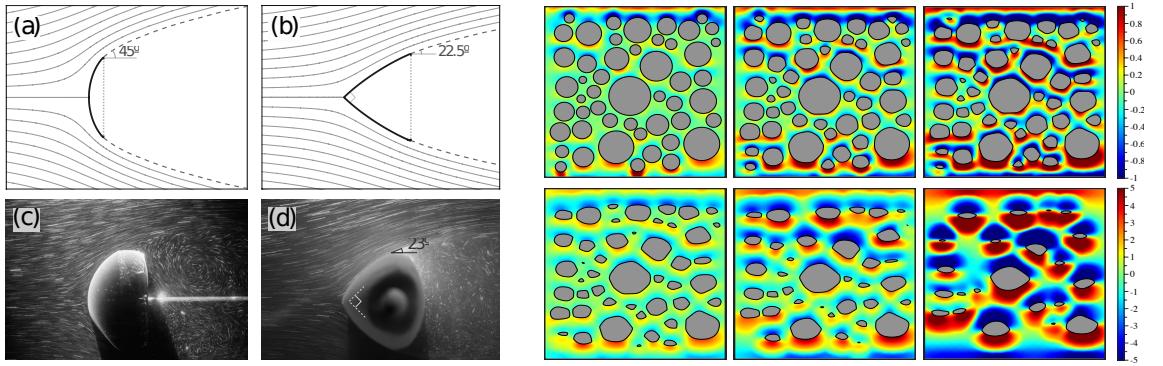


Figure 2: Left panel: (a)–(b) Morphologies formed by a body undergoing dissolution/melting (a) versus one undergoing erosion (b), as predicted by solutions to a singular Riemann-Hilbert problem. (c)–(d) Comparison to laboratory experiments confirms that these solutions accurately predict the emergent shapes. Right panel: Numerical simulations of a porous medium undergoing flow-driven erosion. Color depicts the vorticity field, which directly controls the rate of material removal on each solid surface.

material removal, for instance coast-line or ice-shelf evolution. In these examples, the spatial and temporal complexity of the flows involved present a host of new challenges and opportunities for future research.

Recently, collaborator Bryan Quaife (FSU Scientific Computing) and I extended these ideas to study erosion at the smallest scales. We designed numerical methods to simulate a porous medium eroding due to an internal groundwater flow. As published in the *Journal of Computational Physics* and the *Journal of Fluid Mechanics*, we combine highly accurate boundary-integral methods with stable interface evolution techniques to resolve shape dynamics at the grain scale [23, 6]. An example simulation is shown in the right panel of Fig. 2. As the constituents of the porous medium erode, not only does their size decrease but their shape changes substantially, leading to the appearance of nearly straight channels intervening between the bodies. We find that this channelization process leads to a dramatic reduction in the porous-medium resistivity, much more than could be accounted for by simply the size-reduction of the grains.

## 4 Anomalous waves and extreme events

Rogue waves, though once regarded as figments of seafarers' imagination, have now been recorded in oceans across the globe and pose a recognized danger. These abnormally large waves are generally associated with non-normal statistics and can be caused by a variety of mechanisms. In work with Kevin Speer (FSU oceanography) and then-undergraduate student Tyler Bolles (FSU Mathematics), we designed laboratory experiments to examine the effect of variable bathymetry on wave statistics [3]. These experiments differ from previous ones in two main respects: (1) Our experiments are conducted outside of the deep-water regime so that the well-known rogue-generating Benjamin-Feir instability is absent; (2) Instead of considering gradual variations, our experiments feature abrupt depth changes. We found that this arrangement can indeed generate anomalous wave statistics, with the probability of a rogue wave increasing by a factor of 50 or more. Interestingly, we found these anomalous statistics to be confined to a narrow region downstream of an abrupt depth transition. Within this region, statistics conform remarkably well to a two-parameter gamma distribution, thus providing a clean test for anomalous-wave theories. Since the gamma distribution accurately describes the complete distribution of measured wave statistics, it can be used to predict any statistical feature, for example the moments. This work was published in *Physical Review Fluids*.

In a recent collaboration with Andrew Majda and postdoctoral scholar Di Qi (both at the Courant Institute of Mathematical Sciences), we have developed a theoretical framework to better understand the experimental findings. As published in *PNAS*, the theory is based on statistical mechanics applied to nonlinear wave models and accurately accounts for the skewed distributions observed in the experiments [14, 21].

## 5 Other projects

My research has included several other topics that, for brevity, I will only briefly summarize.

- **Biomechanical models for tumor-induced intracranial pressure:** In collaboration with Harsh Jain (FSU Mathematics), Helen Byrne (Oxford Mathematics) and graduate student Inmaculada Sorribes (FSU), we developed models describing the build-up of inter-cranial pressure due to a growing tumor. This work was recently published in *Biophysical Journal* [26].
- **Membrane formation in hydrothermal vents:** In collaboration with Oliver Steinbock (FSU chemistry) and graduate student Patrick Eastham (FSU Mathematics), we are studying the formation of membranes through reaction/precipitation from a fluid-mechanical perspective [8]. The details of this process may be related to the formation of the first living organisms in primordial hydrothermal vents [7].
- **Ventilation in subterranean caves:** In a recent collaboration with hydrologists and geo-physicists, we analyzed the ventilation patterns of subterranean caves and developed a model to account for the seasonal and synoptic variability [11].
- **Porous-medium convection:** In work with Xiaoming Wang (Fudan University) and graduate student Matthew McCurdy (FSU), we are studying thermal convection in a superposed free-flow/porous-medium using linear and nonlinear stability analysis [15].
- **Complex fluids:** In work with my postdoctoral mentor Michael Shelley (Courant Institute), we developed an efficient computational method to simulate the motion of bodies through a viscoelastic medium. The method exploits a perturbative solution valid in the weak-coupling limit [2].
- **Stratified fluids:** My PhD studies featured the dynamics of stratified fluids, including hydrodynamic stability and fluid-structure interaction [4, 5].
- **Boundary integral methods:** see [1, 23, 9].

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