Constant Intensity Supermodes in Non-Hermitian Lattices

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(Invited Paper)

***Abstract*—We study the existence of a novel class of waves, with constant intensity (CI) in coupled non-Hermitian photonic systems. These generalized plane waves exist only in optical structures that are composed of gain and loss (in both linear and nonlinear domains). In the framework of coupled mode theory, we examine the properties of such supermodes in finite waveguide lattices. In particular, CI-supermodes with periodic, localized, and disordered phases in finite chains of optical elements are considered in detail. Extensions to the nonlinear regime and the connection to the continuum limit are also studied.**

***Index Terms*—PT-symmetry, random lattices, coupled waveguides, non-Hermitian Hamiltonians, coupled mode theory.**

I. Introduction

OVER the past few years, considerable theoretical and experimental effort has been devoted to the new area of parity-time ($\mathcal{PT}$) symmetric photonics. The corresponding non-Hermitian structures combine gain and loss in a unique way with the refractive index satisfying the symmetry relation $n(x) = n^*(-x)$. One of the main features of such non-Hermitian systems is the existence of spontaneous symmetry breaking at an exceptional point (EP) [1]–[4], that is directly related to the novel concept of $\mathcal{PT}$-symmetry which was first suggested in the framework of non-Hermitian quantum mechanics [5]–[7]. The spontaneous symmetry breaking takes the system from a regime of real energy eigenvalues to a partial complex spectrum with conjugate pairs of eigenvalues. Based on these fundamental studies, the idea of $\mathcal{PT}$-symmetry in paraxial waveguide optics was recently introduced [8]–[10] and experimentally realized [11], [12]. These research activities lead to the field of $\mathcal{PT}$ symmetric photonics, with a plethora of theoretical and experimental works spanning a wide range of topics from unidirectional invisibility and solitons to $\mathcal{PT}$-lasers and optical isolators [13]–[28]. Non-optical applications include $\mathcal{PT}$-electronics circuits [29] and $\mathcal{PT}$-acoustic sensor devices [30], [31].

An interesting recent development in this context of non-Hermitian photonics, was the introduction of the idea of constant intensity waves (CI-waves) [28]. The most well-known CI-wave in wave physics is that of simple plane wave, which is a solution of the Helhmoltz equation in a homogeneous (bulk) space. As we will demonstrate, this fundamental concept can be generalized to an inhomogeneous environment by adding gain and loss to the system. This new class of CI-waves exists only for non-Hermitian materials but are not necessarily restricted to $\mathcal{PT}$-symmetric potentials. We have to mention at this point, that besides the importance of these waves for wave optics, waveform engineering and cloaking, they are also of fundamental importance for nonlinear dynamics, since they allow us to study for the first time the phenomenon of modulation instability in an inhomogeneous environment.

The focus of this paper, is to answer the question whether CI-waves also exist in discrete systems. So far they were derived for continuum non-Hermitian Hamiltonians and it was proved that they are radiation eigenmodes of the corresponding waveguide structure [28]. More specifically, in the framework of coupled mode theory, we derive analytical relations for the required gain-loss modulation and the appropriate periodic boundary conditions for such modes to exist. We examine periodic and disordered lattices that support CI-supermodes. Physically speaking, discrete systems can be either evanescently coupled waveguides or optical cavities. Our analysis is general and is valid for both physical setups, and in the continuum limit of infinitely many waveguides the CI-mode of the corresponding non-Hermitian potential [28] is recovered.

II. CI-Waves in Coupled Systems

We begin our analysis by considering optical wave propagation in a non-Hermitian potential in the context of coupled mode theory. In this case, the beam evolution is governed by the following normalized paraxial equation of diffraction for N coupled optical elements (waveguides or cavities)

$$i\frac{dU_n}{dz} + c(U_{n+1} + U_{n-1}) + (\beta_n + ig\gamma_n)U_n = 0 \qquad (1)$$

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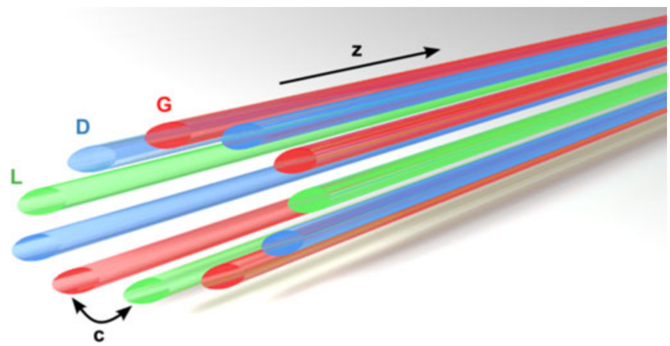

Fig. 1. Schematics of a non-Hermitian lattice of coupled optical waveguides, that supports constant-intensity modes. The waveguides form a ring, because of the imposed periodic boundary conditions and D,L,G,c stands for a dielectric, loss, gain element and for the coupling constant, respectively.

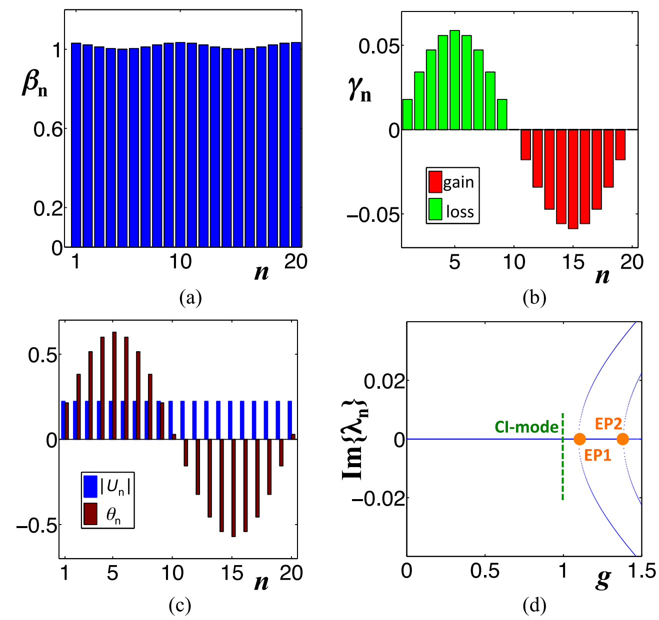

Fig. 2. CI-mode (with $\lambda = 3$) in a lattice of $N = 20$ waveguides for a periodic phase. In particular, (a) real part of the refractive index per waveguide, (b) gain-loss distribution per channel, (c) the amplitude and phase of the CI-supermode, and (d) the imaginary part of all eigenvalues versus g. The exceptional points (EP) are denoted with orange dots and the value of g for which the CI-mode exists with a dashed green line in (d).

where $U_n(z)$ represents the amplitude of the electric field envelope, z is the propagation distance, and $n = 1, ...N$ the waveguide index. c is the coupling coefficient between adjacent neighbors, and taken to be real here [32]. We can normalize the propagation distance in order for c to be equal to one (in normalized units). Each channel is characterized by either gain ($\gamma_n < 0$) or loss ($\gamma_n > 0$) and by its real refractive index β_n. The gain-loss amplitude is described by the parameter g. For $g = 0$ the system is obviously Hermitian. The main question we will address in the most general case of an optical non-Hermitian lattice is if and under which conditions CI-waves exist. We are looking for stationary solutions of constant intensity of the form:

$$U_n(z) = e^{i\theta_n} e^{i\lambda z} \tag{2}$$

where θ_n is a given phase distribution over all waveguide channels and λ is the propagation eigenvalue. It is important to understand that in order for such CI-modes to exist periodic boundary conditions must be imposed at the endpoints of the lattice. In particular, periodic boundary conditions must be valid for the field, namely

$$U_0 = U_N, U_{N+1} = U_1. \tag{3}$$

We can see that in order for the CI-modes given in Eq. (2) to be a solution to Eq. (1), the complex refractive index must satisfy (for any given phase distribution) and gain-loss amplitude $g = 1$ the relations:

$$\beta_n = \lambda - \cos(\theta_{n+1} - \theta_n) - \cos(\theta_{n-1} - \theta_n) \tag{4}$$

$$\gamma_n = -\sin(\theta_{n+1} - \theta_n) - \sin(\theta_{n-1} - \theta_n). \tag{5}$$

Since the CI-waves of Eq. (2) satisfy the periodic boundary conditions of Eq. (3), it follows that the phase distribution θ_n must satisfy the relations:

$$\theta_0 = \theta_N, \theta_{N+1} = \theta_1. \tag{6}$$

Physically speaking, the periodic boundary conditions correspond to an optical ring-lattice of coupled optical elements (waveguides or cavities), as schematically depicted in Fig. 1. The given phase distribution θ_n determines the real and imaginary parts of the refractive index (through Eqs. (4), (5)) whereas the eigenvalue λ (which can be removed by a gauge transformation as it affects only the real part of the index of refraction) is a parameter. CI-waves were first introduced in [28], in the context of the paraxial equation of diffraction and the nonlinear Schrödinger equation in the continuum limit. The CI-waves of such systems were radiation eigenmodes, while in our study here they are true eigenmodes (more precisely supermodes) of the entire system. We also note that for $\lambda = 0$, the CI-mode is unidirectionally invisible, since the wave propagates without any additional phase change and only in one propagation direction (for the opposite direction the complex conjugate potential must be used). Since stationary solutions of constant intensity in the nonlinear lattices are very common in nonlinear optics and dynamics literature, we emphasize that these supermodes exist under linear conditions and are the direct outcome of the non-Hermiticity of the structure.

In order to elucidate our analytical approach with specific results, we consider two numerical examples for the phase distribution θ_n. The first one (Fig. 2) is that of a periodic distribution, namely $\theta_n = 0.6\sin(2\pi n/N)$, with $n = 1, 2, ..., N$. Such a phase results in a lattice with real and imaginary parts presented in Fig. 2(a) and (b), respectively. Even though the Hermitian system ($g = 0$) doesn't support CI-modes the non-Hermitian does (only for $g = 1$), as we can clearly see in Fig. 2(c), where the amplitude and the phase of such a CI-supermode is plotted. Even though the system is not exactly $\mathcal{PT}$-symmetric, we find that the spectrum can be real below a certain value of the gain-loss amplitude. This is indeed the case as is shown in Fig. 2(d), where the imaginary part of all eigenvalues is parametrically plotted against the gain-loss amplitude g. As we can see, the CI-mode exists in the unbroken phase, where the entire spectrum is real.

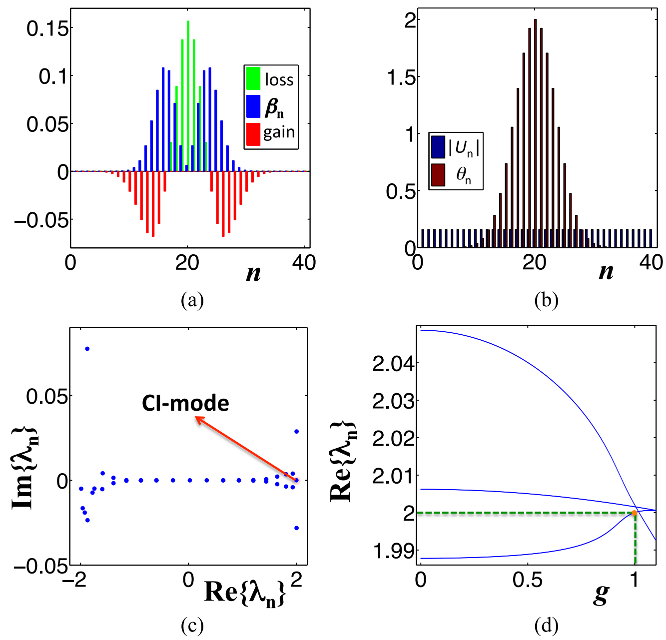

Fig. 3. CI-mode (with $\lambda = 2$) in a lattice of $N = 40$ waveguides for a Gaussian phase. In particular, (a) real part and imaginary parts of the refractive index per waveguide, (b) the amplitude and phase of the CI-supermode, (c) eigenvalue spectrum in the complex plane, and (d) the real part of a few eigenvalues versus g. The value of g for which the CI-mode exists is denoted with a dashed green line in (d).

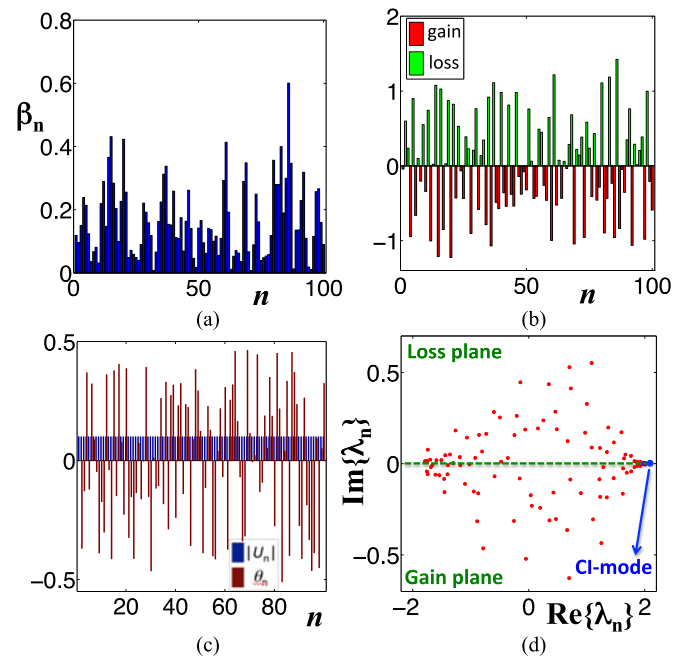

Fig. 4. CI-mode (with $\lambda = 2$, $g = 1$) in a disordered lattice of $N = 100$ waveguides with a random phase. In particular, (a) real part and imaginary parts of the refractive index per waveguide, (b) gain and loss per channel, (c) the amplitude and phase of the CI-supermode, and (d) eigenvalue spectrum in the complex plane. The eigenvalue of the CI-supermode is denoted is denoted with a blue circle.

The second example Fig. 3(a) concerns a localized phase distribution, $\theta_n = 2e^{-((n-N/2)/5)^2}$ for which the CI-mode [Fig. 3(b)] exists in an otherwise complex eigenvalue spectrum [Fig. 3(c)]. The particular lattice is not $\mathcal{PT}$-symmetric but still the eigenvalue of the CI-supermode is, by construction, real. This can be seen in Fig. 3(d), where the real part of the eigenvalues of the first three modes is parametrically plotted versus the gain-loss amplitude g. For $g = 1$, one of the eigenvalues (that of the CI-mode) becomes equal to 2, as is excepted. Both examples show us that we can engineer the refractive index of a lattice by adding gain and loss in order to make one of the modes a CI-wave with a predetermined real eigenvalue λ, that corresponds to the propagation constant of the mode. The rest of the spectrum can be either real (for $\mathcal{PT}$-symmetric matrices) or complex in the general case.

III. CI-Modes in Disordered Systems

One of the most striking results in condensed matter physics pertaining to wave propagation in random media [33]–[35], is the phenomenon of Anderson localization [36]–[40]. In this important field of solid state physics and optics the existence and properties of linear localized modes in random systems has been thoroughly investigated. The majority of the theoretical and experimental studies have however been concentrated on Hermitian media (with the exception of the random lasers literature) where Andreson localization is now well understood. Adding gain and loss to the medium makes the fundamental question of localization generally more complicated [41]. Here we show analytically that any disordered medium that gives rise to Anderson localization (without gain and loss) can also produce extended modes of uniform intensity (CI-supermodes) when a suitable combination of gain and loss is added. For a random and uniformly distributed phase distribution $\theta_n \propto$ rand $[0, 1]$, we can construct for any realization the corresponding non-Hermitian potential that one of its modes is a CI-wave. In particular, we are interested in understanding the spectrum of the following NxN non-Hermitian random matrix:

$$\mathbf{M} = \begin{bmatrix} \beta_1 + ig\gamma_1 & 1 & 0 & \ldots & 1 \\ 1 & \beta_2 + ig\gamma_2 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots\ \beta_{N-1} + ig\gamma_{N-1} & & 1 \\ 1 & 0 & \ldots & 1 & \beta_N + ig\gamma_N \end{bmatrix}.$$

The eigenvalues of $\mathbf{M}$ determine the physical propagation constants of all modes. Such a non-Hermitian eigenvalue problem is obtained by Eq. (1) when we are looking for supermode solutions of the form $U_n(z) = u_n e^{i\mu z}$. One of the modes of this eigenvalue problem $\mathbf{M}\vec{u_n} = \mu\vec{u_n}$, with $\vec{u_n} = [u_1 u_2 \ldots u_N]^\top$, is (by construction) the CI-supermode. In Fig. 4 such a random system of 100 coupled waveguides was considered. The real and imaginary part of the refractive index distribution is depicted for a particular realization of the lattice in Fig. 4(a) and (b), respectively. As we can see, adding gain and loss to such a system can, in principle alter the Anderson localized modes of the Hermitian lattice to extended delocalized modes, one of which is going to be a CI-supermode [Fig. 4(c)] with a real eigenvalue [Fig. 4(d)].

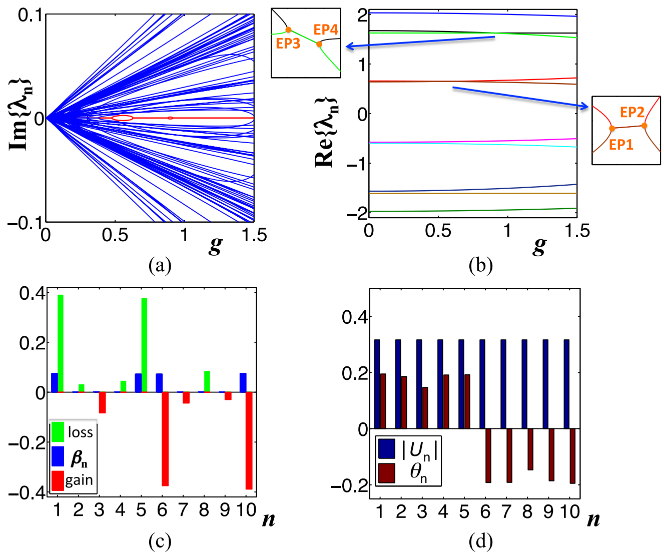

Fig. 5. CI-mode (with $\lambda = 2$) in a disordered lattice of $N = 10$ waveguides for a random phase. In particular, (a) bifurcation curves of the $\max(\mathrm{Im}(\lambda_n)), \min(\mathrm{Im}(\lambda_n))$ versus g for 50 different realizations. The red curve denotes the one realization we consider, (b) parametric plot of the real part of the propagation constants versus g, (c) refractive index distribution per channel, (d) the amplitude and phase of the CI-supermode. For (c), (d) $g = 1$.

One way to avoid the intricate nature and complex spectrum of such a complicated non-Hermitian random matrix, is to impose some symmetries to the lattice (Fig. 5). In particular, we consider the following random matrix, which describes a random $\mathcal{PT}$-symmetric ring lattice:

$$\mathbf{M_{PT}} = \begin{bmatrix} \beta_1 + ig\gamma_1 & 1 & 0 & \ldots & 1 \\ 1 & \beta_2 + ig\gamma_2 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots\ \beta_2 - ig\gamma_2 & & 1 \\ 1 & 0 & \ldots & 1 & \beta_1 - ig\gamma_1 \end{bmatrix}.$$

The real part of the index of refraction β_n is symmetric with respect to an axis of symmetry, whereas the imaginary part γ_n is antisymmetric [15]. In order to get such a $\mathcal{PT}$-symmetric lattice, the phase distribution θ_n must be antisymmetric. Therefore half of the lattice is random and the other half is spatially reflected and conjugated in order for the $\mathcal{PT}$-symmetry to hold. As is well known the $\mathcal{PT}$-symmetry in such random lattices is "exponentially fragile" [15]. This means that for a high number of lattice sites, the symmetry is broken for an exponentially small value of the gain-loss amplitude g. Based on our analysis, a CI-supermode can be found in any random lattice, but since one of our goals is to identify realistic physical systems where the study of modulation instability has a physical meaning (and is not triggered by the existence of amplifying eigenvalues), we are mostly interested in disordered lattices where CI-modes exist in an entirely real eigenvalue spectrum. In Fig. 5(a), for 50 different realizations of a lattice with $N = 10$, the maximum and minimum eigenvalues are shown as a function of g. All of them exhibit an EP for a different value of g. Out of 50 realizations only one (red curve) is unbroken for $g = 1$. For this specific realization we calculate the parametric dependance of the real part of all 10 eigenvalues (top to bottom) to g as plotted in Fig. 5(b). Quite interestingly, the symmetry follows the pattern broken-unbroken-broken-unbroken, due to the multimode nature of the potential. In particular, the second-third and fourth-fifth modes form pairs of complex conjugate eigenvalues after the EPs. For the particular realization chosen here, the corresponding refractive index distribution and the characteristics of the CI-mode, are depicted in Fig. 5(c) and (d), respectively. For higher values of waveguide elements ($N > 20$) the symmetry is broken for extremely low values of the gain-loss amplitude g. Therefore, in order for the CI-mode to occur in a real eigenvalue spectrum (for few of the realizations), we have to consider a rather small number of waveguide channels N. Apart from the direct relevance of these CI-modes to the problem of modulation instability in disordered lattices, one direction of possible interest would also be an extension to z-dependent non-Hermitian Hamiltonians [16] where the refractive index is also modulated along the propagation direction and dynamical localization takes place.

IV. CI-Modes of Discrete Nonlinear Schrödinger Equation

Even though such CI-modes are a linear phenomenon, one natural question, very relevant for modulation instability studies, is if such CI-solutions exist also in nonlinear domain. The answer is positive and we can analytically derive again these nonlinear CI-supermodes. We begin our analysis by considering nonlinear optical wave propagation in a non-Hermitian potential in the context of coupled mode theory. In this case, the beam evolution is governed by the following normalized discrete nonlinear Schrödinger equation for N coupled optical elements (waveguides or cavities)

$$i\frac{dU_n}{dz} + c(U_{n+1} + U_{n-1}) + (\beta_n + i\gamma_n)U_n + \sigma|U_n|^2U_n = 0 \tag{7}$$

where σ represents the sign of the optical Kerr nonlinearity ($\sigma > 0$ for self-focusing and $\sigma < 0$ for defocusing). We are looking for stationary solutions of constant intensity of the following form (for $c = 1$, and $g = 1$):

$$U_n(z) = Ae^{i\theta_n}e^{i\lambda z}. \tag{8}$$

As in the linear case, here also periodic boundary conditions must be satisfied. Substitution of the above equation to the discrete nonlinear Schrödinger equation, leads to the following expression for the real, imaginary part of the lattice and λ, respectively,

$$\beta_n = -\cos(\theta_{n+1} - \theta_n) - \cos(\theta_{n-1} - \theta_n) \tag{9}$$

$$\gamma_n = -\sin(\theta_{n+1} - \theta_n) - \sin(\theta_{n-1} - \theta_n) \tag{10}$$

$$\lambda = A^2\sigma \tag{11}$$

where the nonlinear eigenvalue λ is related to the accumulated self-phase modulation over the propagation distance. Based on such solutions, one can now study the fundamental phenomenon of modulation instability in inhomogeneous non-Hermitian coupled systems, that can have any refractive index distribution, even a random one.

V. Continuum Limit of an Infinite Waveguide Lattice

In this last paragraph, we examine how we can recover the continuum limit, namely the passage from the coupled mode equations Eq. (1) to the paraxial equation of diffraction (and without loss of generality for $\lambda = 0$). This happens when the finite number of waveguides becomes infinite ($N \to \infty$) and when the distance between adjacent channels tends to zero ($\Delta x \to 0$). By applying the gauge transformation $U_n(z) = \phi_n(z)\exp(i2cz)$ Eq. (1) becomes for $g = 1$: $i\frac{d\phi_n}{dz} + c(\phi_{n+1} + \phi_{n-1} - 2\phi_n) + (\beta_n + i\gamma_n)\phi_n = 0$. By allowing $c = 1/(\Delta x)^2$ and taking the limit, the last equation reads, $i\phi_z + \phi_{xx} + V(x)\phi = 0$, with $V(x) = \lim_{\Delta x\to 0}(\beta_n + i\gamma_n)$. The corresponding of equations (4) and (5) (including the $2\phi_n$ extra term) by applying trigonometric identities can be written for the real part, $\beta_n = -2c\cos[(\theta_{n+1} - 2\theta_n + \theta_{n-1})/2]\cos[(\theta_{n+1} - \theta_{n-1})/2] + 2c$ and the imaginary $\gamma_n = -2c\sin[(\theta_{n+1} - \theta_n + \theta_{n-1})/2]\cos[(\theta_{n+1} - \theta_{n-1})/2]$. By using the approximations $\cos x \approx 1 - x^2/2$ for the real part and $\sin x \approx x$, $\cos x \approx 1$ for the imaginary (all of them of second order), we end up with the following limits:

$$\lim_{\Delta x\to 0, N\to\infty} \beta_n = \left(\frac{d\theta}{dx}\right)^2 \quad (12)$$

$$\lim_{\Delta x\to 0, N\to\infty} \gamma_n = -\frac{d^2\theta}{dx^2} \quad (13)$$

where we have used the following approximations for the first $d\theta/dx \approx (\theta_{n+1} - \theta_n)/(\Delta x)$, $d\theta/dx \approx (\theta_n - \theta_{n-1})/(\Delta x)$ and second $d\theta^2/dx^2 \approx (\theta_{n+1} - 2\theta_n + \theta_{n-1})/(\Delta x)^2$ derivative, respectively. By setting $W(x) = d\theta/dx$ we get the continuous non-Hermitian potential of [28] that is $V(x) = W^2 - idW/dx$ and the corresponding CI-solution. From the above derivation, one can understand that the existence of CI-modes is not an artifact of the coupled mode equations, but they exist in both approaches, namely discrete and continuum. In the first case (discrete approach) the CI-wave is a supermode of the lattice whereas in the second one (continuum approach) it is a radiation eigenmode. In both cases the field must satisfy periodic boundary conditions.

VI. Conclusion and Outlook

In summary, we examined the properties of CI-waves in coupled photonic structures. Such modes exist only in the presence of gain and loss and they are part of the eigenvalue spectrum (propagation constants) of the corresponding problem. We derived analytical expressions that are valid for both the linear and the nonlinear domain. The effect of the disorder in such waves was also systematically investigated, with a focus on the transition from Anderson localization (Hermitian system) to uniform intensity eigenmodes (non-Hermitian lattice). Potential applications of such CI-waves may be relevant in scattering geometries. One could be in the context of lasers, where modulation of gain-pump and loss can lead to lasing modes with uniform intensity and lower lasing thresholds. Another direction could be that of directional cloaking-invisibility and acoustics. In order to achieve that one should extend the concept of a CI-wave in an inhomogeneous scattering environment, a topic that we are currently investigating. The acoustic realization would mean that a space could be constructed where an acoustic mode of constant intensity can propagate undistortedly. For one particular frequency, everybody in such a space can hear this frequency with the same intensity.

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