Multicomponent two-dimensional solitons carrying topological charges

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We propose multihump N-component two-dimensional vector solitons for which each constituent carries a different topological charge. These new structures exhibit a unique triple-point phase diagram that is completely absent in the two-component limit.

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Vector solitons are multicomponent solitons that mutually self-trap in a nonlinear medium. Degenerate two-component solitons were first suggested in Kerr nonlinear media[1] and then demonstrated in AlGaAs waveguides.[2] Vector solitons, whose components belong to different modes of their jointly induced potential, were later found in birefringent $\chi^{(3)}$ nonlinear systems.[3] Such composite multimode solitons can have complex structures, and, in many cases, their total intensity profile exhibits multiple humps, as was recently demonstrated experimentally.[4] In general, self-trapping of a multicomponent wave packet occurs when the vector constituents of the wave packet correspond to bound states of their jointly induced waveguide. This is the so-called self-consistency principle, applied to vector solitons.[5] Another necessary requirement for establishing a vector soliton is the absence of any interference (beating) among the eigenmodes of the jointly induced potential. This can be ensured either by use of two orthogonally polarized components[1,3] or by use of two different frequencies when the material has a noninstantaneous nonlinearity, and the frequency difference is larger than the inverse of the material response time.[6] In noninstantaneous nonlinear media, the last requirement is readily satisfied by use of mutually incoherent beams, as has been demonstrated in photorefractives.[7] The recent progress in vector solitons was paralleled by rapid progress in soliton interactions and has stimulated exciting ideas that are unique to multicomponent solitons. Examples include shape transformations[8] and energy exchange between colliding vector solitons.[9]

Here we propose ($N \geq 3$) two-dimensional vector solitons for which each component carries a different topological charge ("spin").[10,11] We find that this family is characterized by a unique triple-point phase diagram, which is completely absent in the two-component case. Moreover, we show that these higher-dimensional vector structures ($N \geq 3$) are composed of lower-dimensional building blocks. With soliton collisions in mind, it is clear that the spin, the multimode nature, and the multihump structure offer new features for interactions between two-dimensional vector solitons. Here we draw on the thresholding nonlinearity[12] and employ the self-consistency principle. However, we emphasize that the core ideas and findings presented here are expected to be universal. As is evidenced by the one-plus-one-dimensional [(1 + 1) D] multimode solitons found for this nonlinearity,[5] the main physical ideas hold for the saturable nonlinearities, as was demonstrated experimentally[4] and theoretically.[13] The composite solitons presented here therefore provide insight into ways to realize these structures in a saturable nonlinearity.

We start from the normalized equations

$$i\,\frac{\partial U_j}{\partial z} + \nabla_\perp^2 U_j + F(I)U_j = 0\,, \tag{1}$$

where U_j, $j = 0, 1, \ldots, N-1$ are the envelopes of the N interacting beams; $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. Equation (1) describes N coupled beams in an optical medium with a normalized refractive-index change $F(I)$ of zero for $I < I_{\mathrm{th}}$ and constant $F_0 = 1$ otherwise, with the total intensity $I = \sum_{j=0}^{N-1} |U_j|^2$. We seek multicomponent soliton solutions to Eq. (1) for which each component U_j carries a different topological charge m_j, in the form

$$U_j(r,\theta,z) = u_j(r)\exp(im_j\theta)\exp(i\mu_j z)\,, \tag{2}$$

where μ_j are the propagation constants of the vector constituents and vanishing boundary conditions at

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infinity are imposed. Substituting Eq. (2) into Eq. (1), we find that

$$\frac{d^2 u_j}{dr^2} + \frac{1}{r}\frac{du_j}{dr} + (\kappa_j^2 - m_j^2/r^2)u_j = 0\,, \qquad (3)$$

where $\kappa_j^2 \equiv F_0 - \mu_j > 0$ and $\mu_j > 0$. The solutions to system (3) are given by

$$u_j(r) = \begin{cases} \eta_j J_{m_j}(\kappa_j r) & 0 \le r \le a \\ \eta_j \dfrac{J_{m_j}(\kappa_j a)}{K_{m_j}(\sqrt{\mu_j}\, a)} K_{m_j}(\sqrt{\mu_j}\, r) & r \ge a \end{cases}, \qquad (4)$$

where J_{m_j} (K_{m_j}) are the regular (modified) Bessel functions of the first (second) kind of order m_j; a is the normalized radius (the so-called V number[14]) of the induced waveguide. The propagation constants satisfy

$$\frac{\sqrt{\mu_j}\, K_{m_j-1}(\sqrt{\mu_j}\, a)}{K_{m_j}(\sqrt{\mu_j}\, a)} + \frac{\kappa_j J_{m_j-1}(\kappa_j a)}{J_{m_j}(\kappa_j a)} = 0\,. \qquad (5)$$

Eigenvalue equation (5) describes the eigenmodes of a weakly guiding step-index fiber for which the polarization states can be found in Ref. 14. We impose the self-consistency condition on the vector components; i.e., the total intensity at the margins of the induced waveguide is equal to the threshold intensity $I_{\mathrm{th}} = \sum_{j=0}^{N-1} \eta_j^2 J_{m_j}^2(\kappa_j a)$. The fundamental two-component configuration was discussed in Ref. 15, and multiple branches of existence curves of composite solitons were found. Here we explore higher-dimensional ($N = 3, 4$) cases and consider multicomponent solitons with $\{m_j\}_{j=0}^{N-1} = (0, 1, 2)$ and $\{m_j\}_{j=0}^{N-1} = (0, 1, 2, 3)$. (Note that degenerate cases such as $(0,1,1)$ and $(0,2,2)$ are also possible.) The reason why we consider here composite solitons in which one component always carries zero charge is that the total intensity vanishes at the center if all components have nonzero charge. Such structures are expected to be highly unstable, even in saturable nonlinearities, as was shown for scalar rings,[16] unless their shapes are azimuthally intensity modulated in the form of necklace beams.[17] By solving Eq. (5) and imposing the self-consistency principle on the N components, we construct a triple-point phase diagram for the $(0,1,2)$ composite solitons [Fig. 1(b)]. Importantly, the existence of this triple-point phase diagram is inherent in $N \ge 3$ composite structures and is absent in the $N = 2$ case. For $\eta_1 = 0$ [fundamental $(0,2)$ vector structure] and by increasing η_2, the total intensity shape changes from single hump (SH) to triple hump (TH) [Figs. 2(a) and 2(b)] with the existance of a relatively small region [the black area in Fig. 1(b)] where solutions correspond to a hollow waveguide [Fig. 2(d)].

When $\eta_1 > 0$, the total intensity still exhibits the same shapes until η_1 reaches 0.8, where a new transition to a double hump (DH) is observed [Fig. 2(c)]. With a further increase in η_1 (decrease in η_2), the existence area of TH shapes shrinks (expands) until it approaches a critical value of $\eta_1^c \approx 1.8$ (and $\eta_2^c \approx 2.3$) where the SH and TH shape boundaries coincide and give rise to a single transition boundary (SH to DH). As η_1 reaches its maximum value 3.57 ($\eta_2 = 0$), we recover the whole range of the existence curve of the basic two-component $(0,1)$ composite structure. Notice that the boundary layer of the hollow waveguide [the darkest area of Fig. 1(b)] has almost a constant width along any cross section in the (η_1, η_2) plane. Conceptually, the $(0,1,2)$ structure can be thought as being made of two fundamental composite states, i.e., $(0,1)$ and $(0,2)$. By superimposing these two basic states we can span the whole range of existence of $(0,1,2)$ vector solitons. Similarly, we can construct a general N-component vector soliton with structure $(0, m_1, m_2, \ldots, m_{N-1})$ from its corresponding basic building blocks $(0, m_j)$. Our last example is the $(0,1,2,3)$ composite structure ($N = 4$). Unlike in the $N = 3$ case for which the phase diagram is two-dimensional, here it is three dimensional, with areas in which soliton solutions exist exhibiting SH, DH, or TH shapes together with solutions that correspond to a hollow waveguide (see Fig. 3).

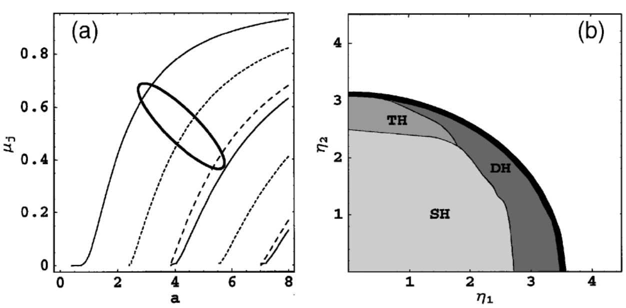

Fig. 1. (a) Propagation constants μ_0 (solid curves), μ_1 (dotted curves), and μ_2 (dashed curves) as a function of the normalized radius a of the induced waveguide (the V number). (b) Triple-point phase diagram for the $(0,1,2)$ composite solitons at $a = 5$.

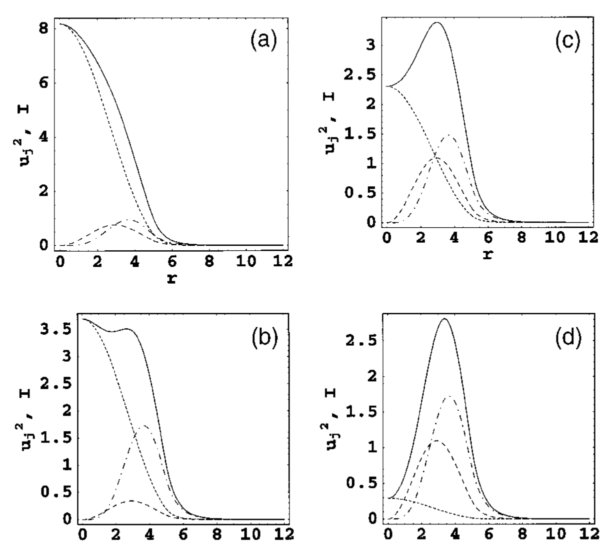

Fig. 2. u_0^2 (dotted curves), u_1^2 (dashed curves), u_2^2 (dotted–dashed curves), and total intensity I (solid curves) for the $(0,1,2)$ case. The parameters (η_1, η_2) are (a) $(1.5, 2)$, (b) $(1, 2.7)$, (c) $(1.8, 2.5)$, and (d) $(1.8, 2.7)$.

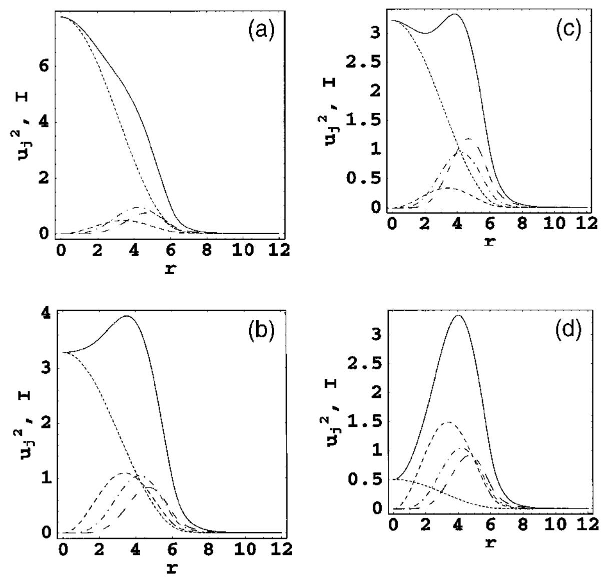

Fig. 3. u_0^2 (dotted curves), u_1^2 (short-dashed curves), u_2^2 (dotted–dashed curves), u_3^2 (long-dashed curves), and total intensity I (solid curves) for the $(0,1,2,3)$ case. The parameters (η_1, η_2, η_3) are (a) $(1.2, 2, 2)$, (b) $(1.8, 2.1, 2.1)$, (c) $(1, 2, 2.5)$, and (d) $(2.1, 2.1, 2.2)$.

Having found these composite solitons, we can reasonably ask: Are they stable or, if they are not, are they at least observable? Because here we deal with a thresholding nonlinearity that does not lend itself to stability analysis, the stability issue is still a fully open question. Nevertheless, we have a good indication that at least the SH and DH families of the composite solitons should be stable, or least weakly unstable, to facilitate experimental observation. First, all saturable nonlinearities support stable (2 + 1)D solitons,[18] so certainly the scalar limit of $u_j = 0$ for $j \geq 1$ is stable. Multimode multihump (1 + 1)D solitons were observed experimentally and were found numerically to be stable for many diffraction lengths.[4] Moreover, recently it was shown that these SH and DH (1 + 1)D solitons are indeed stable over large regions in parameter space.[19] Finally, it is now established that nonlinearity saturation also arrests transverse instabilities of (1 + 1)D solitons in three dimensions, as was found for bright and dark solitons[20] and recently[21] also for the vector solitons reported in Ref. 7. For these reasons, we believe that such structures should be observed in an actual experimental setting.

In conclusion, we have predicted the existence of multihump N-component composite solitons that carry different topological charges, which exhibit a triple-point phase diagram. These multicomponent self-trapped wave packets can provide exciting possibilities for spin interactions, such as spin exchange, spin fragmentation, degeneracy, and multishape transformation.

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