Owen Lewis MAT 21B/Summer Session 2012

Name: _____ Student ID: _____

Quiz 1

Directions: You will have 30 minutes to complete this quiz. Please show all of your work and mark your answers clearly. You may not use any extra resources during the quiz: not your notes, not your book, not a cell phone, not a calculator. Good luck.

1. (6 points) Find the general anti-derivative $\int f(x)\,dx$ when

$$f(x) = \frac{1}{2\sqrt{x}} + 2\sec(x)\tan(x) - 4\cos(x) - e^{-3x}.$$

Solution: We find the general anti-derivative

$$\begin{aligned}
&\int \left(\frac{1}{2\sqrt{x}} + 2\sec(x)\tan(x) - 4\cos(x) - e^{-3x}\right) dx \\
&= \int \frac{1}{2\sqrt{x}}\,dx + 2\int \sec(x)\tan(x)\,dx - 4\int \cos(x)\,dx - \int e^{-3x}\,dx \\
&= \sqrt{x} + 2\sec(x) - 4\sin(x) + \frac{e^{-3x}}{3} + C
\end{aligned}$$

2. (6 points) A bullet is fired at time $t = 1$ with an initial velocity of $v = 400$. If the bullet's acceleration is given by the function $a(t) = -100t$, find the velocity of the bullet as a function of time.
Solution: We know that velocity must be an antiderivative of acceleration. Therefore we begin by finding the general antiderivative of the acceleration.

$$\int a(t)\,dt = \int (-100t)\,dt = -50t^2 + C.$$

Now we use the given initial condition to determine the unknown constant C.

$$v(1) = -50(1^2) + C = 400. \implies C = 450.$$

This fully defines the velocity as a function of time.

$$v(t) = -50t^2 + 450.$$

3. (8 points) From the book, we know that

$$\int_0^b x\,dx = \frac{b^2}{2}, \quad \text{and} \quad \int_0^b x^2\,dx = \frac{b^3}{3}.$$

Use these facts and the properties of definite integrals to calculate

$$\int_1^2 (6x + 3x^2)\,dx.$$

(Do not use the Fundamental Theorem of Calculus, or the Evaluation Theorem)
Solution:

$$\begin{aligned}
\int_1^2 (6x + 3x^2)\,dx &= \int_1^2 6x\,dx + \int_1^2 3x^2\,dx \quad \text{(Rule 4: Sum \& Difference)} \\
&= 6\int_1^2 x\,dx + 3\int_1^2 x^2\,dx \quad \text{(Rule 3: Constant Multiple)} \\
&= 6\left(\int_0^2 x\,dx - \int_0^1 x\,dx\right) + 3\left(\int_0^2 x^2\,dx - \int_0^1 x^2\,dx\right) \text{(Rule 5: Additivity)} \\
&= 6\left(\frac{2^2}{2} - \frac{1^2}{2}\right) + 3\left(\frac{2^3}{3} - \frac{1^3}{3}\right) \quad \text{(From Problem Statement)} \\
&= 6(2 - \frac{1}{2}) + 3(\frac{8}{3} - \frac{1}{3}) \\
&= 12 - 3 + 8 - 1 \\
&= 16
\end{aligned}$$