Owen Lewis MAT 21B/Summer Session 2012

Name: ____________________ Student ID: ____________________

Quiz 2

Directions: You will have 30 minutes to complete this quiz. Please show all of your work and mark your answers clearly. You may not use any extra resources during the quiz: not your notes, not your book, not a cell phone, not a calculator. Good luck.

1. (5 points) Evaluate the definite integral

$$\int_{\pi}^{2\pi} (\sec x + \tan x)^2 \, dx.$$

Solution: We first expand, then use the identity $\tan^2(x) + 1 = \sec^2(x)$.

$$\begin{aligned}
\int_{\pi}^{2\pi} (\sec x + \tan x)^2 \, dx &= \int_{\pi}^{2\pi} \left(\sec^2 x + 2\sec(x)\tan(x) + \tan^2 x\right) \, dx \\
&= \int_{\pi}^{2\pi} \left(2\sec^2 x + 2\sec(x)\tan(x) - 1\right) \, dx \\
&= \left(2\tan(x) + 2\sec(x) - x\right)\Big]_{\pi}^{2\pi} \quad \text{(Evaluation Theorem)} \\
&= (2\tan(2\pi) + 2\sec(2\pi) - 2\pi) - (2\tan(\pi) + 2\sec(\pi) - \pi) \\
&= (0 + 2 - 2\pi) - (0 - 2 - \pi) \\
&= 4 - \pi
\end{aligned}$$

2. (5 points) Find the general anti-derivative

$$\int \sin(3x)e^{\cos(3x)} \, dx$$

Solution: We begin by defining $u = g(x) = \cos(3x)$ and calculating

$$du = \frac{dg}{dx} \cdot dx = -3\sin(3x)dx.$$

This allows us to make the substituion

$$\begin{aligned}
\int \sin(3x)e^{\cos(3x)} \, dx &= \int \left(\frac{-1}{3}\right)(-3\sin(3x))e^{\cos(3x)} \, dx \\
&= \int \left(\frac{-1}{3}\right) e^u \, du \\
&= \frac{-1}{3}e^u + C \\
&= \frac{-1}{3}e^{\cos(3x)} + C.
\end{aligned}$$

3. (5 points) Evaluate the definite integral

$$\int_1^e \frac{2\ln(x)}{x}\,dx$$

Solution: We begin by defining $u = g(x) = \ln(x)$ and calculating

$$du = \frac{dg}{dx}\cdot dx = \frac{1}{x}dx.$$

We also calculate the new limits of integration. The lower limit is $g(1) = \ln(1) = 0$. The upper limit is $g(e) = \ln(e) = 1$. We are now ready to rewrite the integral and evaluate

$$\begin{aligned}\int_1^e \frac{2\ln(x)}{x}\,dx &= \int_1^e 2\ln(x)\frac{1}{x}\,dx\\ &= \int_0^1 2u\,du\\ &= u^2\Big]_0^1\\ &= 1^2 - 0^2\\ &= 1.\end{aligned}$$

4. (5 points) Suppose that we know $G(x) = \int_0^x f(t)\,dt = e^{x^2}$. Use this to calculate $f(1)$. Solution: From the Fundamental Theorem of Calculus, we know that $G'(x) = \frac{dG}{dx} = f(x)$. Therefore we calculate

$$\begin{aligned}f(x) &= \frac{d}{dx}\left[G(x)\right]\\ &= \frac{d}{dx}\left[e^{x^2}\right]\\ &= e^{x^2}\cdot\frac{d}{dx}\left[x^2\right]\\ &= e^{x^2}\cdot 2x\end{aligned}$$

From this we can immediately evaluate $f(1) = 2\cdot 1\cdot e^{1^2} = 2e$.