Owen Lewis MAT 21B/Summer Session 2012

Name: ______________________________ Student ID: ____________________

Quiz 4

Directions: You will have 30 minutes to complete this quiz. Please show all of your work and mark your answers clearly. You may not use any extra resources during the quiz: not your notes, not your book, not a cell phone, not a calculator. Good luck.

1. (6 points) Calculate the indefinite integral

$$\int \left(r^2 + 2r + 1\right) e^r \, dr.$$

Solution: We integrate by parts. Letting $u = r^2 + 2r + 1$, we see that $dv = e^r \, dr$. We then calculate that $v = e^r$ and $du = (2r + 2)\, dr$. Thefore the integration parts allows to write

$$\begin{aligned} \int \left(r^2 + 2r + 1\right) e^r \, dr &= \int u \, dv \\ &= uv - \int v \, du \\ &= \left(r^2 + 2r + 1\right) e^r - \int (2r + 2)\, e^r \, dr. \end{aligned}$$

To calculate this second integral, we use integration by parts a second time. This time, we let $u = 2r + 2$ and we again let $dv = e^r \, dr$. A quick calculation shows that

$$\int (2r + 2)\, e^r \, dr = (2r + 2)\, e^r - \int 2e^4 \, dr = (2r + 2)\, e^r - 2e^r + C.$$

We combine these calculations to show that

$$\int \left(r^2 + 2r + 1\right) e^r \, dr = \left(r^2 + 2r + 1\right) e^r - (2r + 2)\, e^r + 2e^r + C.$$

Simplifying this expression gives $(r^2 + 1)e^r + C$.

2. (7 points) Calculate the definite integral

$$\int_0^{\pi/2} \sin^2(2\theta) \cos^3(2\theta)\, d\theta.$$

Solution: First we use the identity that $\sin^2(x) + \cos^2(x) = 1$ to rewrite the integral

$$\int_0^{\pi/2} \sin^2(2\theta) \cos^3(2\theta)\, d\theta = \int_0^{\pi/2} \sin^2(2\theta) \cos^2(2\theta) \cos(2\theta)\, d\theta = \int_0^{\pi/2} \sin^2(2\theta)(1 - \sin^2(2\theta)) \cos(2\theta)\, d\theta.$$

We now make the substitution $u = \sin(2\theta)$. A calculation shows that $du = 2\cos(2\theta)\, d\theta$. This allows us to calculate the integral

$$\int \sin^2(2\theta) \cos^3(2\theta)\, d\theta = \int \frac{1}{2} u^2 (1 - u^2)\, du = \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) = \frac{1}{2}\left(\frac{\sin^3(2\theta)}{3} - \frac{\sin^5(2\theta)}{5}\right) + C.$$

Finally, we can evaluate the definite integral

$$\int_0^{\pi/2} \sin^2(2\theta) \cos^3(2\theta)\, d\theta = \frac{1}{2}\left(\frac{\sin^3(2\theta)}{3} - \frac{\sin^5(2\theta)}{5}\right)\Bigg]_0^{\pi/2} = \frac{1}{2}\left(\frac{0^3}{3} - \frac{0^5}{5} - \frac{0^3}{3} + \frac{0^5}{5}\right) = 0.$$

3. (7 points) Calcualte the definite integral

$$\int_{1/2}^{1} \frac{y+4}{y^2+y}\, dy.$$

Solution: We wish to expand this integral using the method of partial fractions. We begin by factoring the denominator. To do this, we notice that y is a factor and therefore

$$y^2 + y = y\,(y+1)\,.$$

Because the denominator factors completely into linear terms, with no repeated roots, we can expand the integrand as

$$\frac{y+4}{y^2+y} = \frac{A}{y} + \frac{B}{y+1},$$

where A and B are constants yet to be determined. Clearing the denominator we see that

$$y + 4 = A(y+1) + By.$$

Setting $y = 0$ immediately gives us that $A = 4$, and setting $y = -1$ gives us that $B = -3$. This allows us to rewrite the integral as

$$\int_{1/2}^{1} \frac{y+4}{y^2+y}\, dy = \int_{1/2}^{1} \left(\frac{4}{y} - \frac{3}{y+1}\right) dy = \left(4\ln|y| - 3\ln|y+1|\right)\Big]_{1/2}^{1}.$$

Evaluating this gives

$$\begin{aligned}
\left(4\ln|y| - 3\ln|y+1|\right)\Big]_{1/2}^{1} &= 4\ln(1) - 4\ln(1/2) - 3\ln(2) + 3\ln(3/2) \\
&= 4\cdot 0 + 4\ln(2) - 3\ln(2) + 3\left(\ln(3) - \ln(2)\right) \\
&= 3\ln(3) - 2\ln(2) \\
&= \ln(27) - \ln(4) \\
&= \ln\left(\frac{27}{4}\right).
\end{aligned}$$