Math 5110/6830

Homework 1
Due: Thursday Sept. 11 (In Class)

A: (Suggested) Read Chapters 1 and 2 in the textbook.

1. Consider the following linear model for the population of boars in a swamp

$$\begin{bmatrix} b_{n+1} \\ B_{n+1} \end{bmatrix} = \vec{p}_{n+1} = \begin{pmatrix} 0 & m \\ \sigma & r \end{pmatrix} \cdot \begin{bmatrix} b_n \\ B_n \end{bmatrix} = A \cdot \vec{p}_n. \tag{1}$$

Here, B_n represents the number of adult boars, while b_n represents the number of immature 'boarlettes'.

   (a) Interpret the 3 parameters m, σ, and r in your own words. What values are reasonable for each?

   (b) Find the eigenvalues of the matrix A.

   (c) Now assume that $r = \sigma = 1/2$. Find the value of m which ensures that the population of boars does not die off. Interpret this result in words.

2. We now consider a model of the same boar population, but with human imposed harvesting

$$\vec{p}_{n+1} = \begin{pmatrix} 0 & m \\ \sigma & r \end{pmatrix} \cdot \vec{p}_n - \vec{H}, \tag{2}$$

where $\vec{H} = \begin{bmatrix} h_b \\ h_B \end{bmatrix}$ encodes the harvest of boarlettes (h_b) and adults (h_B).

   (a) Using on your calculations from Problem 1b, do you believe there will be a positive equilibrium population (hypothesize)? If so, under what conditions?

   (b) Calculate the equilibrium population of juvenile and adult boars (note: you should not be assuming anything about r or σ).

   (c) Under what conditions is there a positive equilibrium population? Interpret your condition in words. Is it the same as the condition that you hypothesized in problem 2a.

3. Consider the following discrete model of medication in the blood stream of a patient,

$$M_{n+1} = M_n - f(M_n) \cdot M_n + S. \tag{3}$$

Here, M_n is the concentration of the medication on day n, S is the concentration of medicine administered in a daily dosage, and $f(M_n)$ is the fraction of medication absorbed by the body. Suppose that

$$f(M) = \frac{M^\alpha}{K^\alpha + M^\alpha}, \tag{4}$$

where K and α are also parameters. Assume that $K = 2$ and $S = 1$. In addition to your calculations, use `cobweb.m` (provided on the course web page) to generate cobweb plots and answer the following questions:

(a) Find find the *positive* equilibrium value.

(b) For which values of α does the equilibrium seem stable. Why? Include a cobweb diagram of an illustrative case.

(c) For which values of α does the concentration M_n oscillate towards the equilibrium. Why? Include a cobweb diagram of an illustrative case.

(d) Are there values of α for which the equilibrium is unstable? What are they? Why? Include a cobweb diagram of an illustrative case.