Math 5110

Midterm 3
Due: 4pm Tuesday Dec. 15th (In Owen's office, or Owen's mailbox in JWB)

Read these instructions carefully:

This takehome exam is to be completed alone. You may not consult with your classmates or anyone else. If you have any questions, direct them to Owen Lewis. You may use your notes, your book, Maple and/or Matlab. You may use Pplane to help generate plots and determine stability. However, you will be graded on your explanations. Please thoroughly describe and interpret whatever the computer is telling you if you want full credit. Do not turn in your Maple or Matlab code.

The exam is due to Owen Lewis by 4pm Dec. 15th, 2014. Please bring it directly to LCB 301, or put it in Owen's mailbox in JWB. The mailbox room closes **before 4pm**, so *plan ahead*. No late exams will be accepted.

Problems (just one):

1. We will analyze a simple model of a chemical oscillator. This system is known as the "Brusselator" and is named after the scientist who proposed it. It is often used as a very simple "proxy" for a physiological calcium oscillator in cellular dynamics. In dimensionless form, the equations are

$$\begin{aligned} \frac{dx}{dt} &= 1 - (b+1)x + x^2 y, \\ \frac{dy}{dt} &= bx - x^2 y. \end{aligned} \tag{1}$$

   Here, x and y are the dimensionless concentrations of two chemical species. The single parameter b is assumed to be non-negative.

   (a) Find all equilibria of the system.

   (b) Sketch the null clines and then use this information to help construct a "trapping region" in the phase plane. Make sure you also use the equations to justify that your region is a trapping region.

   (c) Now determine the stability of all fixed points that you found. Identify any parameter values at which a bifurcation occurs.

   (d) Identify the types of any bifurcations that you found in the previous section. How can you tell?

   (e) Putting all this information together, what are all possible behaviors of the Brusselator that may be observed depending on b?