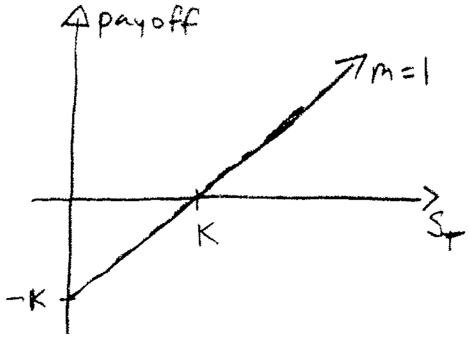
Derivative Markets - Module 2

Section 1: Put-Call Parity

The payoffs for the following compound positions are the same.

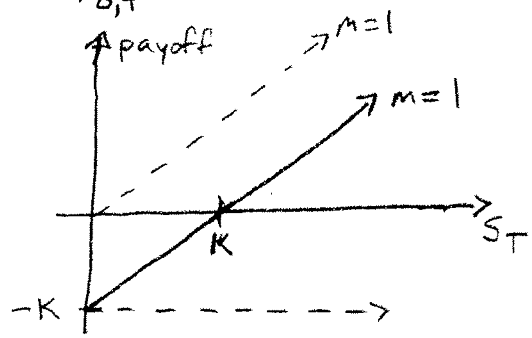
1. Long Call(K,T) + Short Put(K,T)

$IC = Call(K,T) - Put(K,T)$



2. Long Pre-paid Forward + Short Bond(K)

$IC = F_{0,T}^{P} - Kv^{T}$



Since the payoffs of the two positions are the same, in order to avoid an arbitrage opportunity, the initial costs of the two positions must be the same. We get

$$Call(K,T) - Put(K,T) = F_{0,T}^{P} - Kv^{T}$$

Remarks:

1. This is the most general form of an important equation in Derivative Markets that is referred to as **Put-Call Parity**. A special case of this general form that you are likely to encounter is when the options are at-the-money ($K = S_0$) and the underlying asset does not pay dividends ($F_{0,T}^{P} = S_0$). In this case,

$$Call(S_0,T) - Put(S_0,T) = S_0 - S_0 v^{T}$$

2. We can rewrite the Put-Call Parity equation to get other equivalent equations, which in turn describe other equivalent compound positions. For example, by rearranging the terms we can rewrite the general Put-Call Parity equation as

$$Call(K,T) + Kv^{T} = Put(K,T) + F_{0,T}^{P}$$

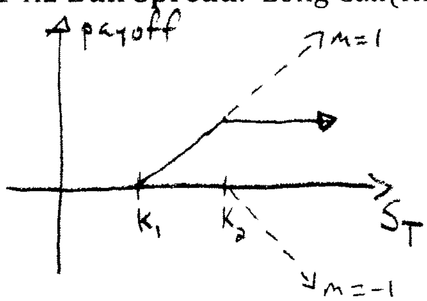
which implies that a [Long Call(K,T) + Long Bond(K)] position is equivalent to a [Long Put(K,T) + Long Pre-paid Forward] position. You can check this by graphing payoff diagrams for each position.

Section 2: Directional Positions

A **directional position** is a compound position which becomes profitable when the spot price at expiration of the underlying asset moves in one directions (either higher or lower), but becomes unprofitable when the spot price at expiration moves in the other direction. An investor is **bullish** on an asset if the investor believes the asset will increase in value, whereas an investor is **bearish** on an asset if the investor believes the asset will decrease in value.

Examples: (K1<K2)

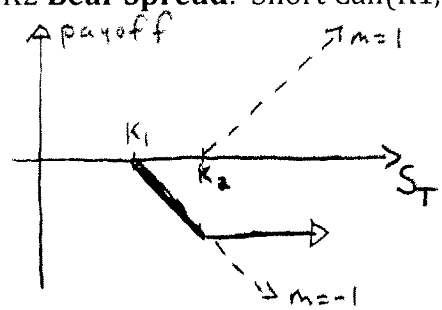
1. Long K1-K2 **Bull Spread**: Long Call(K1,T) + Short Call(K2,T)



$$IC = Call(K_1,T) - Call(K_2,T)$$

Remark: Long Put(K1,T) + Short Put (K2,T) is another way to form a long K1-K2 bull spread.

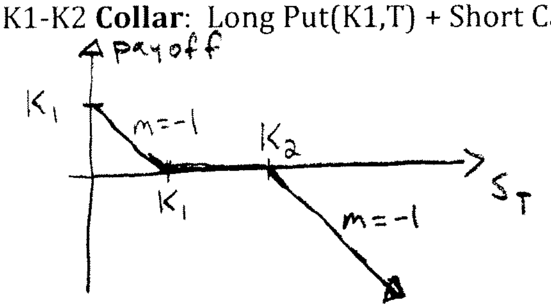
2. Long K1-K2 **Bear Spread**: Short Call(K1,T) + Long Call(K2,T)



$$IC = -Call(K_1,T) + Call(K_2,T)$$

Remark: Short Put(K1,T) + Long Put (K2,T) is another way to form a long K1-K2 bear spread.

3. Long K1-K2 **Collar**: Long Put(K1,T) + Short Call(K2,T)



$$IC = Put(K_1,T) - Call(K_2,T)$$

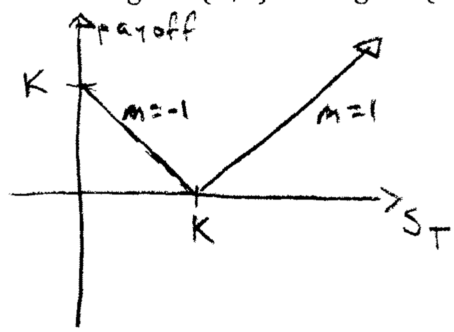
Remark: The **collar width** equals K2 – K1. A **zero-cost collar** is a collar with IC = 0; i.e. Put(K1,T) = Call(K2,T)

Section 3: Volatility Positions

A **volatility position** is a compound position in which profitability will depend on how much the spot price at expiration varies from the center of the position.

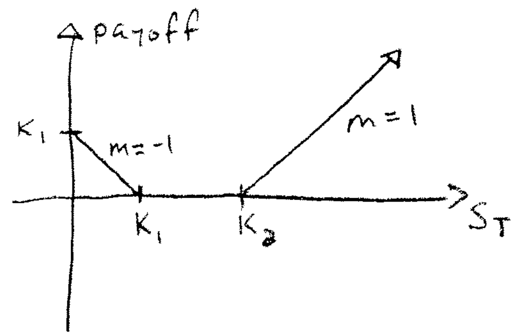
Examples: (K1<K2<K3)

1. Long **Straddle**: Long Call(K,T) + Long Put(K,T)



$$IC = Call(K,T) + Put(K,T)$$

2. Long K1-K2 **Strangle**: Long Put(K1,T) + Long Call(K2,T)

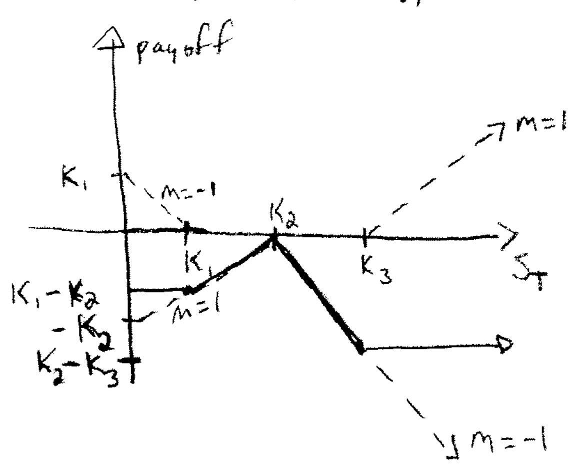


$$IC = Put(K_1,T) + Call(K_2,T)$$

3. Long K1-K2-K3 **Butterfly Spread**:
   This position can be achieved in many ways. The method shown below is sometimes referred to as purchasing an **insured written straddle**.

   Short K2 Staddle + Long K1-K3 Strangle: (In terms of Calls and Puts, this is)
   [Short Call(K2,T) + Short Put(K2,T)] + [Long Put(K1,T) + Long Call(K3,T)]

$$IC = -Call(K_2,T) - Put(K_2,T) + Put(K_1,T) + Call(K_3,T)$$



Section 4: Futures

A **futures** contract is basically an exchange-traded forward contract. In the example that follows, we'll illustrate the type of question that you may see on the FM Exam 2, as well as introduce notation and terminology associated with futures. Notable differences between futures and forwards are:

1. Whereas forwards are settled at expiration, futures are settled daily by **marking to market**. (An example follows.)

2. Since they're settled daily, futures are liquid, which means one can easily enter into a reverse position to cancel an existing position. Daily settlement also reduces credit risk, which is the risk that the party that owes money fails to make a payment.

3. Futures contracts are standardized and traded in exchanges, whereas forward contracts are **over-the-counter** (between individuals and not traded on a market). Forward contracts can be customized to fit the needs of buyers and sellers.

4. Futures markets have **daily price limits**, which is a move in the futures price that triggers a temporary halt in trading.

Example: The specifications of a futures contract are:

1. The underlying asset is the (hypothetical) C&J Index.
2. The size of each contract is $250 x C&J Index.
3. Settlement is weekly by marking-to-market.
4. The **margin** is 10% of the **notional value**.
5. The **maintenance margin** is 80%.
6. The risk-free interest rate is 6% compounded continuously.

Suppose we are currently in January, the current September C&J Index futures price is 1000, and we go long two September C&J Index futures contracts. Determine the margin balance after 1, 2, and 3 weeks if the corresponding September C&J futures prices at these times are 1010, 980, and 975, respectively.

Solution to Example:

Since the current September C&J Index futures price is 1000, and the size of each contract is $250 x C&J Index, the notional value of each contract is 250,000. Therefore the notional value for this position is 500,000. The initial margin is 10% of 500,000, which is 50,000. This is the amount we need to put up in order to open the contract. It earns interest at the risk-free rate. Through marking to market, as we will illustrate below, the margin balance changes over time. If the margin balance ever falls below the maintenance margin of 40,000 (80% of 50,000), then in order to keep the contract open, we must add a deposit to bring the margin balance back to the initial margin. We can perform the marking to market accounting as follows:

Week	Futures Price	Change in Price	Margin Balance
0	1000	-	$B_0 = 50000$
1	1010	+10	$B_1 = B_0 e^{\frac{.06}{52}} + 2(250)(+10)$ $B_1 \approx 55058$
2	980	-30	$B_2 = B_1 e^{\frac{.06}{52}} + 2(250)(-30)$ $B_2 \approx 40121$
3	975	-5	$B_3 = B_2 e^{\frac{.06}{52}} + 2(250)(-5)$ $B_3 \approx 37668$

The balance in the margin account at time one is determined by first adjusting the balance at time 0 with interest. Since there are 2 contracts, the size of each contract is 250 times the index, and the index changed by +10 from time 0 to time 1, we adjust the margin balance by another 2(250)(+10) = 5000. Similar adjustments are made at subsequent times as illustrated in the above table.

Note that at time 3, the balance in the margin account is 37668, which is less than the maintenance margin. Therefore, a **margin call** will be made at time 3. We are required to add a deposit of 50000 – 37668 = 12332 in order to keep the position open. Otherwise the position will be closed and we will be refunded the balance of 37668.

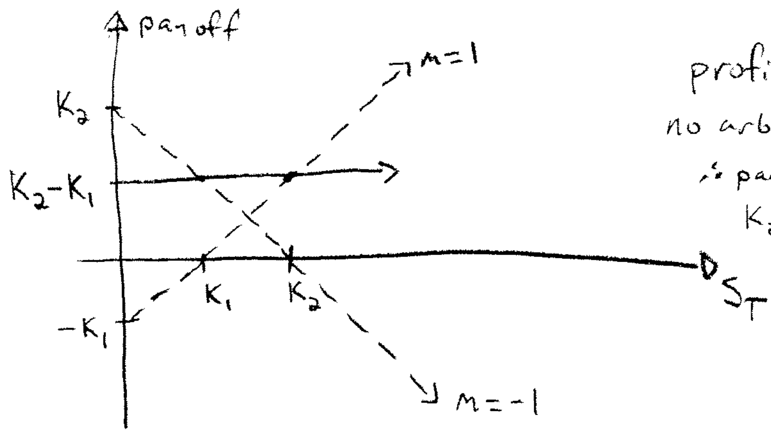
Section 5: Box Spreads, Ratio Spreads, and Swaps

A **box spread** is achieved by using options to create a synthetic long forward at one price and a synthetic short forward at another price. The resulting payoff of these forwards is constant, and so it will not depend on the spot price at expiration of the underlying asset. Thus it is said to have *no asset price risk.*

Example: ($K1 < K2$)

[Long Call(K1,T) + Short Put(K1,T)] + [Short Call(K2,T) + Long Put(K2,T)]

$$IC = Call(K_1,T) - Put(K_1,T) - Call(K_2,T) + Put(K_2,T)$$



profit = payoff − AVIC

no arbitrage ⇒ profit = 0

∴ payoff = AVIC

$K_2 - K_1 = AVIC$

$\therefore IC = (K_2-K_1)v^T$

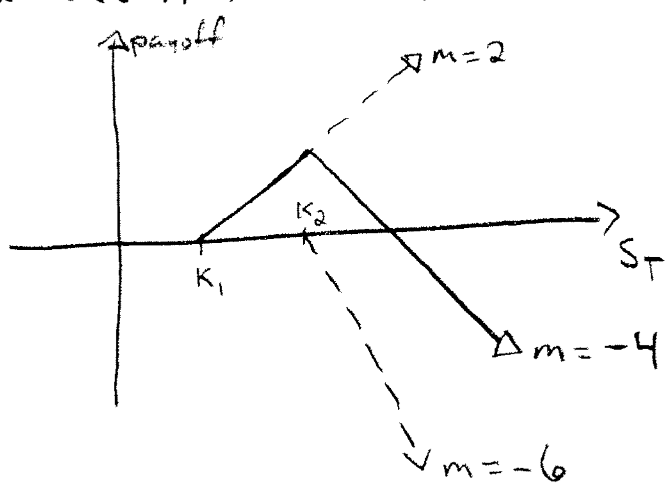
(Also follows from Put-Call Parity)

A **ratio spread** is achieved by buying *m* calls at one strike price and selling *n* calls at a different strike price. We can choose *m* and *n* such that the initial cost of the ratio spread is 0, and when this is done we call it a **paylater** strategy.

Example: ($K1 < K2$)

2 Long Call(K1,T) + 6 Short Call(K2,T)

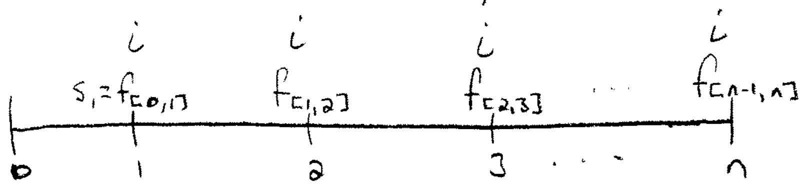
$$IC = 2Call(K_1,T) - 6\,Call(K_2,T)$$



A **swap** is a contract that exchanges one set of payments for another set of payments such that the present values of the two sets of payments are equal. The life of the swap is called the **swap term** (or **swap tenor**). Commonly, a set of non-level payments is exchanged for a set of level payments, and in this case the amount of the level payments is called the **swap price**. A **pre-paid** swap would occur if the original set of payments is exchanged for a single payment at time 0.

An **interest rate swap** is an agreement in which the forward rates that are implied by the current term structure of interest rates are exchanged for a constant set of forward rates, called the **swap rate,** such that, per dollar invested at the beginning of each period, the two interest streams will produce the same (time 0) present value. The timeline is:

s_k = k-year spot rate ; $f_{[k,k+1]}$ = forward rate from $t=k$ to $t=k+1$



$$PV_1 = PV_2$$

$$PV_1 = i \cdot v_{s_1} + i\, v_{s_2}^2 + i\, v_{s_3}^3 + \cdots + i\, v_{s_n}^n$$

$$PV_2 = s_1 v_{s_1} + f_{[1,2]} \cdot v_{s_2}^2 + f_{[2,3]} \cdot v_{s_3}^3 + \cdots + f_{[n-1,n]} v_{s_n}^n$$

Recall that $(1+s_k)^k = (1+s_{k-1})^{k-1}(1+f_{[k-1,k]})$

$$= \frac{s_1}{1+s_1} + \left(\frac{(1+s_2)^2}{1+s_1} - 1\right) \cdot \frac{1}{(1+s_2)^2} + \left(\frac{(1+s_3)^3}{(1+s_2)^2} - 1\right) \frac{1}{(1+s_3)^3} + \cdots + \left(\frac{(1+s_n)^n}{(1+s_{n-1})^{n-1}} - 1\right) \cdot \frac{1}{(1+s_n)^n}$$

$$= \underbrace{\frac{s_1}{1+s_1} + \left(\frac{1}{1+s_1}\right.}_{=1} - \left.\frac{1}{(1+s_2)^2}\right) + \left(\frac{1}{(1+s_2)^2} - \frac{1}{(1+s_3)^3}\right) + \cdots + \left(\frac{1}{(1+s_{n-1})^{n-1}} - \frac{1}{(1+s_n)^n}\right)$$

$$= 1 - \frac{1}{(1+s_n)^n} = 1 - v_{s_n}^n$$

$$\therefore\ PV_1 = PV_2 \implies i = \frac{1 - v_{s_n}^n}{v_{s_1} + v_{s_2}^2 + v_{s_3}^3 + \cdots + v_{s_n}^n}$$

Section 6: Lagniappe (Extra)

Lease Rate and **Implied Repo Rate**
Sometimes the underlying asset is such that it doesn't make sense to say that the asset pays dividends. In this case, what was previously referred to as the continuous dividend rate, δ, is called the **lease rate** and what was previously referred to as the risk-free interest rate, r, is called the **implied repo rate**. The difference between the implied repo rate and the lease rate is $\alpha = r - \delta$, called the **cost-of-carry**.

An **outright purchase** of a stock means one pays for and takes ownership of the stock today. A **fully leveraged purchase** of a stock means one takes ownership of the stock today, but does not pay for the stock until a later time, T.

A **convertible bond** is a bond that, under certain circumstances, pays the holder in stock instead of cash coupons and/or redemption value. A **mandatorily convertible bond** is a bond that *always* pays the holder in stock instead of cash.

Reasons to Use Derivatives:

(1) Risk Management (Hedging and Insurance)
(2) Speculation
(3) Reduce Transaction Costs
(4) Regulatory Arbitrage (Circumvent Regulations and Avoid Taxes)

Types of Risk:
Diversifiable Risk refers to risk that can be shared; for example, the risk of one facing financial ruin due to having to pay for their house to be rebuilt after the house burns down can be shared by all homeowners by having the homeowners purchase fire insurance on their houses. Risks that cannot be shared are called **non-diversifiable risks**. For example, the risk of one facing financial ruin due to a stock market crash cannot be shared among all the investors in the stock market, since the crash would financially affect all the investors.

Perspectives on Derivatives:

(1) Economic Observer
(2) End-User
(3) Market Maker

Buying and Selling in Financial Markets:
When buying, an investor will pay the market maker the **ask** price for the derivative, and when selling, an investor will be paid the **bid** price from the market maker for the derivative. Taken together, this is called the **bid-ask spread**.

Derivative Markets Module 2 Problems

1. A stock is currently trading for 50. The risk-free interest rate is 6% annual effective. A 50-strike call option on the stock with 2 years until expiration is currently trading for 9. Determine the premium for a 2-year 50-strike put option on the stock.

2. A stock that pays continuous dividend at a rate of 2% is trading at 100. A 110-strike put option on the stock with 6 months until expiration is currently trading for 15 whereas the premium for the corresponding 6-month 110-strike call option on the same stock is 6.18. Determine the continuously compounded risk-free interest rate consistent with these prices.

3-11 For Numbers 3 – 11, you are given the following premium information for 1-year call and put options on a non-dividend paying stock that is currently priced at 50. The risk-free interest rate is 3% compounded continuously.

Strike Price	Call Premium	Put Premium
48	6.84	3.42
49	5.95	3.50
50	5.16	3.68
51	4.58	4.07
52	4.22	4.68
53	4.11	5.54

3. Ed buys a 50-52 bull spread using calls. Determine Ed's profit if the spot price at expiration is 51.

4. Nancy buys a 50-52 bull spread using puts. Determine Nancy's profit if the spot price at expiration is 51.

5. Carol buys a 50-52 bear spread using calls. Determine Carol's profit if the spot price at expiration is 51.

6. Greg buys a 49-51 bear spread. Determine the maximum profit and maximum loss on Greg's position at expiration.

7. Linda buys a 48-52 collar. Determine the collar width, the break-even point, the maximum profit, and the maximum loss on Linda's position.

3-11 (Repeated from above) For Numbers 3 – 11, you are given the following premium information for 1-year call and put options on a non-dividend paying stock that is currently priced at 50. The risk-free interest rate is 3% compounded continuously.

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8. Judy sells a 50-strike straddle. Determine the range of spot prices at expiration for which Judy will profit from the position.

9. Christopher purchases a 49-52 strangle. Determine the range of spot prices at expiration for which Christopher will profit from the position.

10. Jason enters into a butterfly spread position by insuring a short 50-strike straddle by buying a 49-51 strangle. Determine Jason's break-even point(s), his maximum profit, and his maximum loss.

11. A ratio spread with paylater strategy is established by buying three 48-strike calls and selling *n* 52-strike calls. Determine the profit if the spot price at expiration is 55.

12. A 2-year box spread on an underlying asset is established by creating a short synthetic forward with forward price of 80 and a long synthetic forward with a forward price of 90. Assuming a 2% annual effective risk-free interest rate, determine the initial cost of this box spread.

13. The specifications of a futures contract are:

    1. The underlying asset is a bushel of wheat. Contract Size = 1000 bushels
    2. Settlement is weekly by marking-to-market.
    3. The margin is 10% of the notional value and the maintenance margin is 80%.
    4. The risk-free interest rate is 4% compounded continuously.

    Suppose the futures price for September wheat is 9.98 per bushel, and we buy three such futures contracts. If at any time the balance in the margin account is below the maintenance margin, a sufficient amount is deposited to keep the contract open. After 1, 2, and 3 weeks, the corresponding futures prices for a bushel of September wheat are 9.80, 9.72, and 9.30, respectively. Determine the amount needed at the end of week 3 to keep the contract open.

14-18 For Numbers 14 and 18, you are given the following:

The annual yield rate on 1-year zero coupon bonds is 3.0%.
The annual yield rate on 2-year zero coupon bonds is 4.5%.
The annual yield rate on 3-year zero coupon bonds is 5.5%.
The annual yield rate on 4-year zero coupon bonds is 6.0%.

14. An investor wishes to swap payments of 1000, 1250, 1500, and 1750 at times 1, 2, 3, and 4, respectively. Determine the prepaid swap price using the above term structure of interest rates.

15. Based on past yields, a wheat farmer believes he will produce 10000 bushels of wheat each year for the next 4 years. You are given:

    The 1-year forward price of wheat is 9.75 per bushel.
    The 2-year forward price of wheat is 9.95 per bushel.
    The 3-year forward price of wheat is 10.05 per bushel.
    The 4-year forward price of wheat is 10.15 per bushel.

    Determine the 4-year swap price based upon this information.

16. Determine the 3-year swap rate consistent with the given term structure of interest rates.

17. Determine the 4-year swap rate consistent with the given term structure of interest rates.

18. Determine the swap rate for a 1-year deferred 3-year interest rate swap that is consistent with the given term structure of interest rates.