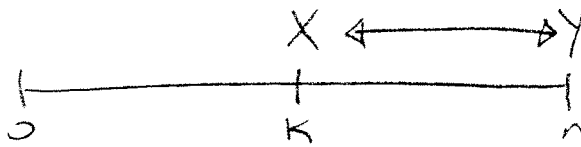# Module 1

Section 1: Accumulation Functions

The **accumulation function**, denoted $a(t)$, gives the value at time t for an initial time 0 investment of 1. So $a(0) = 1$. The **amount function**, denoted $A(t)$, gives the value at time t for an initial time 0 investment of C. So $A(0) = C$. The amount function is obtained from the accumulation function by multiplying the accumulation function by the amount of the initial investment. The "timeline" is:
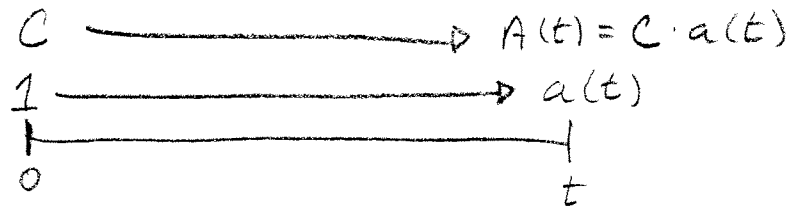
We focus primarily on the accumulation function. We **accumulate** from time 0 to time t by multiplying by $a(t)$, and we **discount** from time t back to time 0 by dividing by $a(t)$. We can relate a time k value of X to its equivalent (or indifference) time n value Y using accumulation functions as follows:

$$Y = X \cdot \frac{a(n)}{a(k)}, \text{ or solving for } X \text{ we get, } X = Y \cdot \frac{a(k)}{a(n)}$$

For each deposit, the **amount of interest** earned between times k and n equals the difference between the equivalent time n value of the deposit and the equivalent time k value of the deposit.

Module 1 Section 1 Problems:

Note: The problems at the end of each section are "warm-up" exercises. These are generally **not** the type of problem that you will see on an actuarial exam. Actuarial exam type problems are generally harder problems that cover more than what is covered in any one section. These types of problems are at the end of each module. As with all math problems, strive to use correct notation.

For Problems 1-8, you are given the following accumulation function information:

$$a(1) = 1.2, \ a(2) = 1.5, \ a(3) = 2.0, \text{ and } a(4) = 3.0$$

(Remember that $a(0) = 1$ for all accumulation functions.)

1. 100 is deposited at time $t = 0$. Determine the accumulated amount at time $t = 3$.

2. Determine the present value at time $t = 0$ of 60 at time $t = 4$.

3. The value at time $t = 2$ is 300. Determine the accumulated value at time $t = 3$.

4. Determine the discounted value at time $t = 1$ of a value of 600 at time $t = 4$.

5. Given 480 at time $t = 1$, plus 300 at time $t = 3$:

    a. Determine the (total) present value at time $t = 0$ of the two payments.
    b. Determine the (total) accumulated value at time $t = 4$ of the payments.
    c. Determine the (total) value at time $t = 2$ of the payments

6. Determine the amount of interest earned from time $t = 2$ to time $t = 4$ if 500 is invested at time $t = 0$.

7. Determine the amount of interest earned from time $t = 2$ to time $t = 3$ if 300 is invested at time $t = 1$.

8. Determine the amount of interest earned from time $t = 2$ to time $t = 4$ if 240 is invested at time $t = 1$ and an additional 300 is invested at time $t = 3$.

Answers to Module 1 Section 1 Problems

1) 200

2) 20

3) 400

4) 240

5) (a) 550

  (b) 1650

  (c) 825

6) 750

7) 125

8) 450