Section 2: Simple and Compound Interest

Simple Interest:

$a(t) = 1 + it$, where i is the simple interest rate and t is measured in years

Discrete Compound Interest: (Converted and Payable are synonyms for Compounded)

$a(t) = (1 + i)^t$, where i is the periodic effective interest rate (eir) and t is measured in same time unit (match periods for i and t).

In the context of discrete compounding, $1 + i$ is the periodic accumulation factor and $v = \frac{1}{1+i}$ is the periodic discount factor.

Continuously Compounding Interest:

$a(t) = e^{\delta t}$, where δ is the continuously compounded interest rate and t is measured in years. Actuaries refer to δ as the (constant) force of interest.

In the context of continuous compounding, e^{δ} is the annual accumulation factor and $v = e^{-\delta}$ is the annual discount factor.

Periodic Effective Interest Rates:

$i_k = \frac{a(k)-a(k-1)}{a(k-1)}$ is the periodic effective interest rate (eir) for the k^{th} period.

In the context of compounding, i_k is constant. We abbreviate the monthly effective interest rate by meir, the quarterly effective interest rate is abbreviated by qeir, etc.

Nominal Interest Rates:
In the context of discrete compounding, we let m denote the number of compounding periods per year. The nominal interest rate is the rate quoted and is denoted by $i^{(m)}$. The periodic eir is $i = \frac{i^{(m)}}{m}$.

Equivalent Rates: (Indifference Rates)

When compounding, we determine equivalent rates by accumulating or discounting a given amount (we can use \$1) over an arbitrary period of time. For simple interest, we must be given the period of time over which to accumulate or discount.

Module 1 Section 2 Problems:

1. An account pays 3% simple interest.
    a. Determine the accumulation function.
    b. Determine the effective interest rate for the 4^{th} year.
    c. Determine the effective interest rate for the 6^{th} year.
    d. Determine the effective interest rate for the 9^{th} month.
    e. Determine the effective interest rate for the 13^{th} month.

2. Four years ago David made an initial deposit into an account that pays 6% simple interest. Unfortunately, David does not remember how much he initially deposited into the account. He currently has 1240 in the account. Determine how much David will have in the account one year from now.

3. An account pays 6% interest, compounded monthly.
    a. Determine the accumulation function.
    b. Determine the effective interest rate for the 3^{rd} month.
    c. Determine the effective interest rate for the 5^{th} month.
    d. Determine the monthly accumulation factor.
    e. Determine the monthly discount factor.
    f. Determine the effective interest rate for the 2^{nd} quarter.
    g. Determine the effective interest rate for the 4^{th} quarter.
    h. Determine the quarterly accumulation and discount factors.

4. Four years ago David made an initial deposit into an account that pays 6% interest, compounded semiannually. Unfortunately, David does not remember how much he initially deposited into the account. He currently has 1240 in the account. Determine how much David will have in the account one year from now.

5. An account pays interest using a constant force of interest equal to 7%.
    a. Determine the accumulation function.
    b. Determine the annual accumulation and discount factors and the aeir.
    c. Determine the monthly accumulation and discount factors and the meir.

6. An account pays 7% interest compounded annually. Determine the equivalent force of interest.

7. An account pays 4% interest compounded semiannually. Determine the equivalent force of interest.

8. Four years ago David made an initial deposit into an account that pays interest using a constant force of interest equal to 6%. Unfortunately, David does not remember how much he initially deposited into the account. He currently has 1240 in the account. Determine how much David will have in the account one year from now.

9. X is deposited into an account that pays 3% interest, compounded quarterly. The accumulated value after 8 years is 10000. Determine X.

10. 200 is deposited into an account that pays 6% simple interest for the first 3 years the money is in the account, then 8% compounded quarterly for the next 2 years the money is in the account, then a constant force of interest of 5% thereafter. Determine the amount in the account after 10 years.

Answers to Module 1 Section 2 Problems:

1) (a) $a(t) = 1 + .03t$ t - years

(b) $i_4 \doteq 2.7529\%$

(c) $i_6 \doteq 2.609\%$

(d) $i_{9^{th}\,month} \doteq 0.2451\%$

(e) $i_{13^{th}\,month} \doteq 0.24279\%$

2) $X = 1300$

3) (a) $a(t) = 1.005^t$ t - months

(b) & (c) $i_3 = i_5 = .005$

(d) $maf = 1.005$

(e) $mdf = v_{.005} = \frac{1}{1.005}$

(f) & (g) $j = qeir \doteq .01508$

(h) $qaf = 1 + j \doteq 1.01508$ $qdf = v_j \doteq \frac{1}{1.01508}$

4) $X = 1315.52$

5) (a) $a(t) = e^{.07t}$ t - years

(b) $aaf = e^{.07}$ $adf = v_i = e^{-.07}$ $aeir = e^{.07} - 1 = i$

(c) $maf = e^{.07/12}$ $mdf = v_j = e^{-.07/12}$ $meir = e^{.07/12} - 1 = j$

6) $\delta = \ln(1.07)$

7) $\delta = 2\ln(1.02)$

8) $X = 1316.68$

9) $X = 7873.33$

10) $X = 355.05$