Section 3: Simple and Compound Discount

Simple Discount:

$a(t) = (1 - dt)^{-1}$, where d is the simple discount rate and t is measured in years

Discrete Compound Discount: (Converted and Payable are synonyms for Compounded)

$a(t) = (1 - d)^{-t}$, where d is the periodic effective discount rate (edr) and t is measured in same time unit (period).

In the context of discrete compounding, $(1 - d)^{-1}$ is the periodic accumulation factor and $v = 1 - d$ is the periodic discount factor.

Continuously Compounding Discount: (Same as continuously compounded interest.)

$a(t) = e^{\delta t}$, where δ is the continuously compounded discount rate and t is measured in years.

In the context of continuous compounding, e^{δ} is the annual accumulation factor and $v = e^{-\delta}$ is the annual discount factor.

Periodic Effective Discount Rates:

$d_k = \frac{a(k) - a(k-1)}{a(k)}$ is the periodic effective discount rate (edr) for the k^{th} period.

In the context of compounding, d_k is constant. We abbreviate the monthly effective discount rate by medr, the quarterly effective discount rate is abbreviated by qedr, etc.

Nominal Discount Rates:
In the context of discrete compounding, we let m denote the number of compounding periods per year. The nominal discount rate is the rate quoted and is denoted by $d^{(m)}$. The periodic edr is $d = \frac{d^{(m)}}{m}$.

Equivalent Rates: (Indifference Rates) (Same as with interest rates.)

When compounding, we determine equivalent rates by accumulating or discounting a given amount (we can use \$1) over an arbitrary period of time. For simple discount, we must be given the period of time over which to accumulate or discount.

Module 1 Section 3 Problems:

1. An account pays interest using a 5% simple discount rate.
    a. Determine the accumulation function.
    b. Determine the effective interest rate for the 4^{th} year.
    c. Determine the effective discount rate for the 4^{th} year.
    d. Determine the effective interest rate for the 6^{th} year.
    e. Determine the effective discount rate for the 6^{th} year.

2. Four years ago Carol made an initial deposit into an account that pays interest using a 6% simple discount rate. Unfortunately, Carol does not remember how much she initially deposited into the account. She currently has 1000 in the account. Determine how much Carol will have in the account one year from now.

3. An account pays interest using a 8% discount rate, compounded semiannually.
    a. Determine the accumulation function.
    b. Determine the semiannual discount and accumulation factors.
    c. Determine the equivalent aeir and aedr.

4. Four years ago Carol made an initial deposit into an account that pays interest using a 6% discount rate, compounded quarterly. Unfortunately, Carol does not remember how much she initially deposited into the account. She currently has 1000 in the account. Determine how much Carol will have in the account one year from now.

5. An account pays interest using a constant force of discount equal to 7%.
    a. Determine the accumulation function.
    b. Determine the annual accumulation and discount factors.
    c. Determine the equivalent aeir and aedr.

6. An account pays interest using a 7% discount rate, compounded annually. Determine the equivalent force of interest.

7. An account pays interest using a 4% discount rate, compounded semiannually. Determine the equivalent force of interest.

Answers to Module 1 Section 3 Problems:

1) (a) $a(t) = (1 - .05t)^{-1}$ t - years

(b) $i_4 = \frac{5}{80}$

(c) $d_4 = \frac{5}{85}$

(d) $i_6 = \frac{5}{70}$

(e) $d_6 = \frac{5}{75}$

2) $X = 1085.71$

3) (a) $a(t) = (.96)^{-t}$ t - semiannual periods

(b) $sdf = .96 = v$ $saf = v^{-1} = .96^{-1}$

(c) $aeir = i = .96^{-2} - 1$ $aedr = d = 1 - (.96)^2$

4) $X =$ ~~1177.38~~ 1062.32

5) (a) $a(t) = e^{.07t}$ t - years

(b) $aaf = e^{.07}$ $adf = v = e^{-.07}$

(c) $aeir = i = e^{.07} - 1$ $aedr = 1 - e^{-.07}$

6) $\delta = -\ln(.93)$

7) $\delta = -2\ln(.98)$