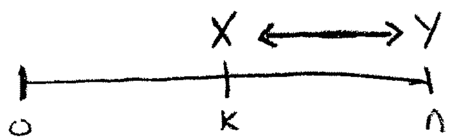
Section 4: General Force of Interest

Relating force of interest to accumulation functions:

Given $a(t)$, then $\delta_t = \frac{a'(t)}{a(t)}$ (t is measured in years)

Given δ_t, then $a(t) = e^{\int_0^t \delta_r \, dr}$ (t is measured in years)

Accumulating and Discounting using General Force of Interest:



$$Y = X \cdot e^{\int_k^n \delta_t \, dt}, \text{ or equivalently, } X = Y \cdot e^{\int_n^k \delta_t \, dt}$$

Special Cases:

1.

$$\delta_t = c \cdot \frac{f'(t)}{f(t)} \Longrightarrow a(t) = \left(\frac{f(t)}{f(0)}\right)^c$$

2. Constant Force of Interest: $\delta_t = \delta$ (see earlier notes on continuous compounding)

$$a(t) = e^{\delta t}$$

Module 1 Section 4 Problems:

1. Given $a(t) = 1 + 2t + \frac{1}{2}t^2$, determine an expression for the general force of interest.

2. Given $a(t) = 100 + 200t + 50t^2$, determine δ_2.

3. Given $\delta_t = \frac{6t}{2+6t^2}$ determine $a(1)$.

4. Suppose $\delta_t = .02t, t > 0$.
    a. Determine the accumulation function.
    b. Determine the accumulated value at time 7 of the time 3 value of 100.

5. Given $\delta_t = \frac{.03}{1-.03t}$ determine the discounted value at time 2 of the time 6 value of 50.

Solutions to Module 1 Section 4 Problems:

1) $\delta_t = \dfrac{2+t}{1+2t+\frac{1}{2}t^2}$

2) $\delta_2 = \frac{4}{7}$

3) $a(1) = 2$

4) (a) $a(t) = e^{.01t^2}$

(b) $X \doteq 149.18$

5) $X \doteq 43.62$