Section 2: Basic Annuity Formulas

Key Formulas for Valuing Geometric Sums and Series:

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}$$

$$1 + r + r^2 + \cdots = \frac{1}{1 - r}$$

VEP means to *value each payment at the valuation date.* The number of terms in the VEP expression will equal the number of payments. The closed rule formulas (CRF's) follow from the VEP expressions using the above key formulas for valuing geometric sums and series.

Value Each Payment (VEP) and Closed Rule Formulas (CRF) for Basic Level Annuities

$$a_{\overline{n}|} \overset{VEP}{=} v + v^2 + \cdots + v^n \overset{CRF}{=} \frac{1 - v^n}{i}$$

$$\ddot{a}_{\overline{n}|} \overset{VEP}{=} 1 + v + v^2 + \cdots + v^{n-1} \overset{CRF}{=} \frac{1 - v^n}{d}$$

$$s_{\overline{n}|} \overset{VEP}{=} 1 + (1+i) + (1+i)^2 + \cdots + (1+i)^{n-1} \overset{CRF}{=} \frac{(1+i)^n - 1}{i}$$

$$\ddot{s}_{\overline{n}|} \overset{VEP}{=} (1+i) + (1+i)^2 + \cdots + (1+i)^n \overset{CRF}{=} \frac{(1+i)^n - 1}{d}$$

$$a_{\overline{\infty}|} \overset{VEP}{=} v + v^2 + \cdots \overset{CRF}{=} \frac{1}{i}$$

$$\ddot{a}_{\overline{\infty}|} \overset{VEP}{=} 1 + v + v^2 + \cdots \overset{CRF}{=} \frac{1}{d}$$

Module 2 Section 2 Problems:

For Numbers 1-10, first draw the timeline (include the valuation date) for each given cash flow. Then check your work by comparing your timeline to the to the timeline given in the corresponding Numbers 1-10 of the previous section. Then compute the numeric value of the annuity at the valuation date, using the given interest assumption.

1. Determine the accumulated value of a 3-year annuity due with annual payments of 4 using an annual effective interest rate of 5%.

2. Determine the present value of a 15-month annuity immediate with monthly payments of 7 using a monthly effective interest rate of 0.6%

3. An annuity pays 8 at the end of each year for 5 years, starting at the end of the 12^{th} year. Determine the value of the annuity immediately before the first payment using an annual effective interest rate of 7%.

4. An annuity pays 3 semiannual payments of 4. Determine the present value of the annuity 1 year before the first payment using a nominal interest rate of 10% compounded semiannually.

5. An annuity consists of 4 biannual payments of 5. Determine the present value of the annuity 1 year before the first payment using an annual effective interest rate of 3%.

6. Determine the present value, one month before the first payment, of a perpetuity consisting of quarterly payments of 2, using an interest rate of 8% compounded quarterly.

7. A 7-year annuity due with annual payments has a first payment of 10, a second payment of 20, a third payment of 10, a fourth payment of 20, etc. Determine the accumulated value of the annuity immediately after the last payment, using an annual effective interest rate of 6%.

8. An annuity with monthly payments has 4 payment of 12 followed by two months without payments followed by two more months with payments of 12. Determine the value of the annuity one month after the 4^{th} payment of 12, using a nominal rate of 12% compounded monthly.

9. For the same annuity as in Number 8, determine the present value of the annuity two months before the first payment using $i^{(12)} = 9\%$.

10. Determine the accumulated value of a 10-year annuity immediate with annual payments of 6, where interest is credited using 3% per annum for the first 5 years and 4% per annum thereafter.

11. Determine the present value of a 10-year annuity immediate with monthly payments of 30 using an annual effective interest rate of 6%.

12. Determine the present value of a perpetuity due with quarterly payments of 60 using a nominal interest rate of 12.18% compounded semiannually.

Answers to Module 2 Section 2 Problems

1) $4\ddot{s}_{\overline{31}|.05} = 13.2405$

2) $7a_{\overline{151}|.006} = 100.1268367$

3) 35.09769005

4) 10.37427821

5) 17.80857871

6) 101.3289279

7) 119.8307923

8) 72.62269449

9) 69.2858683

10) 71.25418831

11) 2721.729649

12) 2060