Section 5: Geometric Annuities

A sequence of terms forms a geometric progression if there is a "common ratio" between consecutive terms of the sequence. This means that given any term in the sequence, we can get the next term by multiplying by the common ratio.

A **geometric annuity** is an annuity for which the payments form a geometric progression. There are no special actuarial annuity symbols for the values of geometric annuities. When valuing geometric annuities, since the payments form a geometric progression, then the VEP expression for the value of the annuity will be a geometric sum with common ratio $r > 0$. We can determine the value of a geometric annuity using the following 3-step process.

Step 1: VEP
Step 2: Factor out the first term
Step 3: Recognize basic level VEP expressions and use TVM

After we factor out the first term of the VEP expression, as stated in Step 2, the second factor in the resulting expression will be a geometric sum that looks like $1 + r + r^2 + \cdots$ where the number of terms of the sum equals the number of payments of the annuity. For Step 3, we recognize the following facts:

1. If $r < 1$, think of $r = v = \frac{1}{1+i}$ and the sum is $1 + v + v^2 + \cdots \overset{VEP}{=} \ddot{a}_{\overline{n}|i}$ where n is the number of payments and $i = \frac{1}{r} - 1$.
2. If $r > 1$, think of $r = 1 + i$ and the sum is $1 + (1+i) + (1+i)^2 + \cdots \overset{VEP}{=} s_{\overline{n}|i}$ where n is the number of payments and $i = r - 1$.

Summarizing, for Step 3, we recognize that $1 + r + r^2 + \cdots = \begin{cases} \ddot{a}_{\overline{n}|\left(\frac{1}{r}-1\right)} & \text{if } r < 1 \\ s_{\overline{n}|(r-1)} & \text{if } r > 1 \end{cases}$

Geometric Annuity Immediate: In the special case of determining the present value of a geometric annuity immediate with periodic payments of $K, K(1+j), \cdots$, we can use the following formula, where i is the periodic effective interest rate:

$$PV = K\frac{1 - \left(\frac{1+j}{1+i}\right)^n}{i - j}$$

Geometric Perpetuity: If the annuity is a perpetuity, then the geometric sum representing its present value is actually a convergent geometric series with common ratio $r < 1$. We determine the present value by using the fact that the geometric series converges to the ratio of the first term to $(1 - r)$; that is,

$$PV = \frac{\text{First Term}}{1 - r}$$

Module 2 Section 5 Problems:

1. Determine the value of the sum $1 + 1.02 + 1.02^2 + \cdots + 1.02^{19}$

2. Determine the value of the sum $1 + 0.97 + 0.97^2 + \cdots + 0.97^{23}$

3. Determine the value of the sum $1.04 + 1.04^2 + \cdots + 1.04^{14}$

4. Determine the value of the sum $0.95 + 0.95^2 + \cdots + 0.95^{36}$

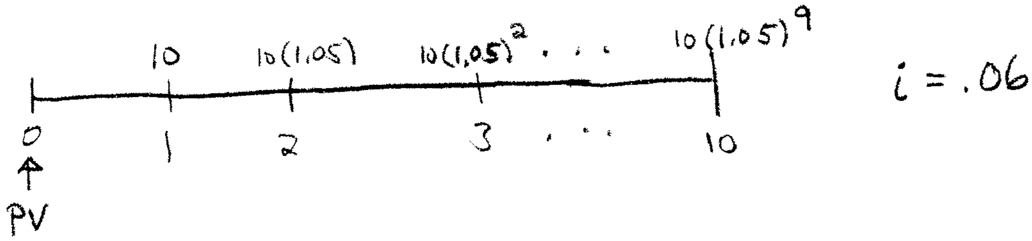
5. Determine the value of the sum $1 + \left(\frac{1.03}{1.05}\right) + \left(\frac{1.03}{1.05}\right)^2 + \cdots + \left(\frac{1.03}{1.05}\right)^{59}$

6. Determine the value of the sum $\left(\frac{1.0506}{1.03}\right) + \left(\frac{1.0506}{1.03}\right)^2 + \cdots + \left(\frac{1.0506}{1.03}\right)^{20}$

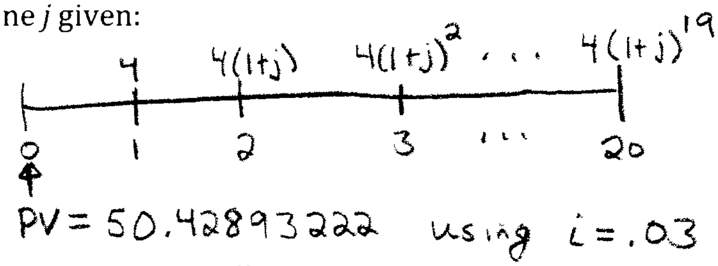
7. Determine the value of the series $1.07(0.98) + 1.07(0.98)^2 + 1.07(0.98)^3 \cdots$

For Numbers 8-15, determine the value, at the given valuation date and using the given periodic effective interest rate, of the cash flow shown.

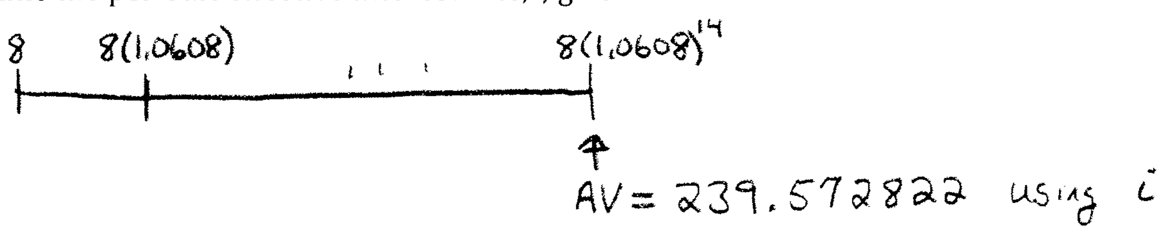
8.



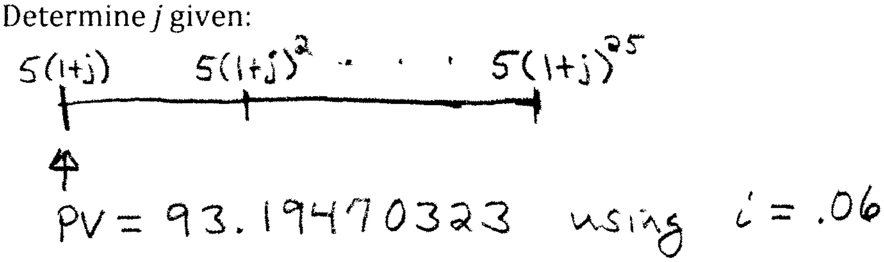
9. Determine j given:

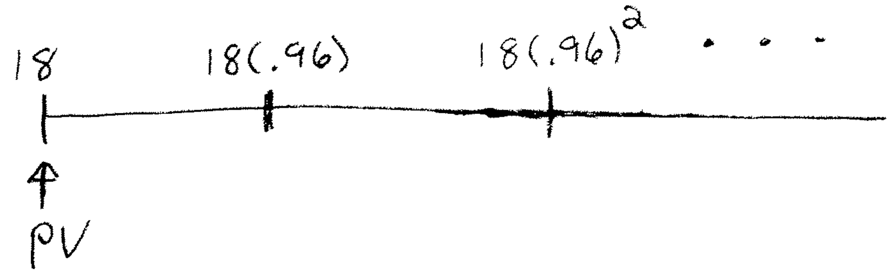


10. Determine the periodic effective interest rate, i, given:



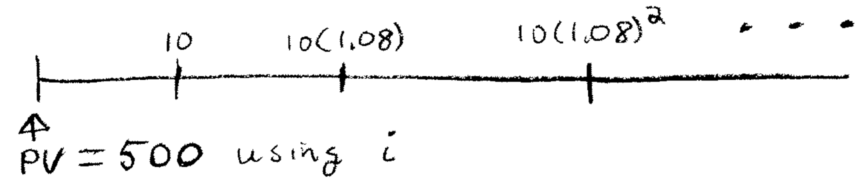
11. Determine j given:



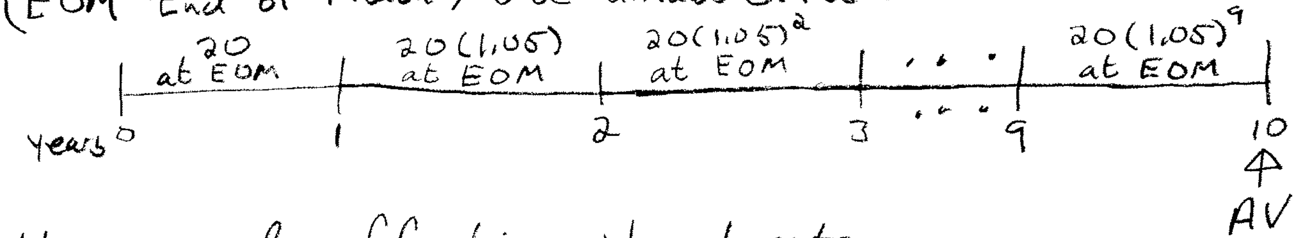
12. 

$i = .05$

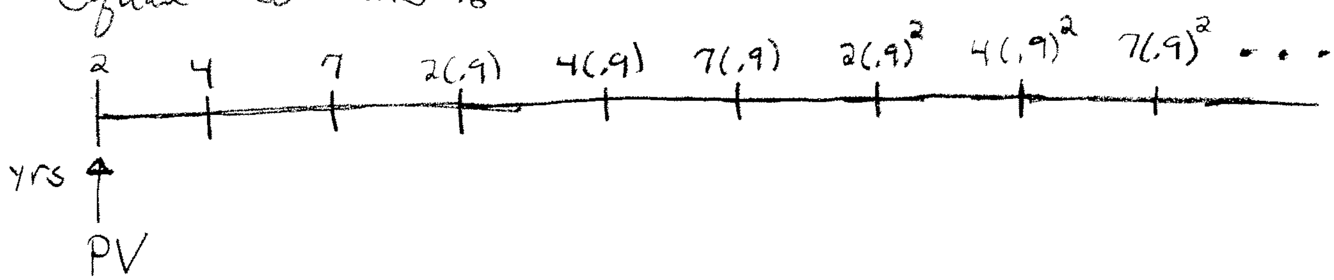
13. Determine the periodic effective interest rate *i*, given:



14. (EOM - End of Month) Use annual effective interest rate = 6%



15. Use annual effective interest rate equal to 2.5%



Answers to Module 2 Section 5 Problems

1) 24.2973698
2) 17.2860926
3) 19.02358764
4) 16.00219492
5) 35.94097196
6) 24.78331719
7) 52.43
8) 90.43374826
9) -0.02
10) 0.04
11) 0.03
12) 210
13) 0.10
14) 3992.638209
15) 76.4952