Section 8: Summary of Annuity Formulas

You have now been baptized into the world of actuarial notation. Consider all the annuity symbols we've seen and/or discussed in this module:

$a_{\overline{n|}i}$ $\ddot{a}_{\overline{n|}i}$ $s_{\overline{n|}i}$ $\ddot{s}_{\overline{n|}i}$ $a^{(m)}_{\overline{n|}i}$ $\ddot{a}^{(m)}_{\overline{n|}i}$ $s^{(m)}_{\overline{n|}i}$ $\ddot{s}^{(m)}_{\overline{n|}i}$

$(Ia)_{\overline{n|}i}$ $(I\ddot{a})_{\overline{n|}i}$ $(Is)_{\overline{n|}i}$ $(I\ddot{s})_{\overline{n|}i}$ $(Da)_{\overline{n|}i}$ $(D\ddot{a})_{\overline{n|}i}$ $(Ds)_{\overline{n|}i}$ $(D\ddot{s})_{\overline{n|}i}$

$(Ia)^{(m)}_{\overline{n|}i}$ $(I\ddot{a})^{(m)}_{\overline{n|}i}$ $(Is)^{(m)}_{\overline{n|}i}$ $(I\ddot{s})^{(m)}_{\overline{n|}i}$ $(Da)^{(m)}_{\overline{n|}i}$ $(D\ddot{a})^{(m)}_{\overline{n|}i}$ $(Ds)^{(m)}_{\overline{n|}i}$ $(D\ddot{s})^{(m)}_{\overline{n|}i}$

$(I^{(m)}a)^{(m)}_{\overline{n|}i}$ $(I^{(m)}\ddot{a})^{(m)}_{\overline{n|}i}$ $(I^{(m)}s)^{(m)}_{\overline{n|}i}$ $(I^{(m)}\ddot{s})^{(m)}_{\overline{n|}i}$ $(D^{(m)}a)^{(m)}_{\overline{n|}i}$ $(D^{(m)}\ddot{a})^{(m)}_{\overline{n|}i}$

$(D^{(m)}s)^{(m)}_{\overline{n|}i}$ $(D^{(m)}\ddot{s})^{(m)}_{\overline{n|}i}$ $\bar{a}_{\overline{n|}i}$ $\bar{s}_{\overline{n|}i}$ $(I\bar{a})_{\overline{n|}i}$ $(I\bar{s})_{\overline{n|}i}$ $(\bar{I}\bar{a})_{\overline{n|}i}$ $(\bar{I}\bar{a})_{\overline{n|}i}$

$(D\bar{a})_{\overline{n|}i}$ $(D\bar{s})_{\overline{n|}i}$ $(\bar{D}\bar{a})_{\overline{n|}i}$ $(\bar{D}\bar{a})_{\overline{n|}i}$

For the OVERWHELMING majority of problems you'll see on the exam, you will only need the first four basic annuity symbols in the first row, and the eight basic increasing/decreasing annuity symbols in the second row (12 total symbols). On rare occasions you may see the four basic upper *m* annuity symbols in the first row, and possibly the two basic level continuous annuity symbols in the last row (another 6 symbols, for a total of 18). It is very unlikely that you'll see any of the other symbols and, although the material is on the syllabus for the exam, you may safely omit these from your studies.

For each of the 18 annuity symbols to be focused on from the last paragraph, you should know what payments are being valued and the location of the valuation date. With this knowledge, the VEP formulas should be intuitive. For CRF's, note that all the other CRF's can be easily derived from the following three basic CRF's as discussed below.

$a_{\overline{n|}} = \frac{1-v^n}{i}$ $(Ia)_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}}-nv^n}{i}$ $(Da)_{\overline{n|}} = \frac{n-a_{\overline{n|}}}{i}$

We derive the other CRF's by knowing how to "accumulate" and "decorate" the above three formulas. For example, if we want *s* instead of *a* then we "accumulate" the above formula by multiplying by $(1+i)^n$. If we want double-dot, then we replace the periodic effective interest rate *i* in the denominator by its equivalent periodic effective discount rate *d*. If we want upper *m* then we replace the periodic effective rate in the denominator with its equivalent upper *m* nominal rate. For example, from the basic *Da* formula, we have $(D\ddot{s})^{(m)}_{\overline{n|}i} = \frac{n-a_{\overline{n|}}}{d^{(m)}} \cdot (1+i)^n = \frac{n\cdot(1+i)^n - s_{\overline{n|}}}{d^{(m)}}$ (although this is one I said you can omit!)

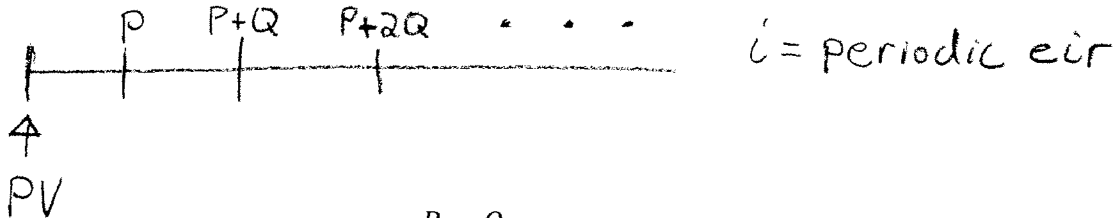
Perpetuities

Note that there is no such thing as the accumulated value of a perpetuity (think about it). Except for arithmetic perpetuities, the present values of all the perpetuities in this module are convergent geometric series. Therefore we can use the following general fact, where r represents the common ratio of the geometric series:

$$PV = \frac{\text{First Term}}{1 - r}$$

This is the basic fact used to derive CRF's such as $a_{\overline{\infty}|} = \frac{1}{i}$ and $\ddot{a}_{\overline{\infty}|} = \frac{1}{d}$ and others.

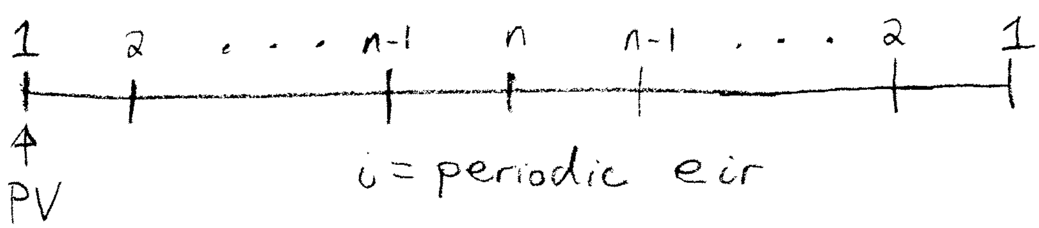
For arithmetic perpetuities, it is worthwhile to commit to memory the following formula for the present value of a perpetuity with initial payment of P and common difference of Q, where the valuation date is one payment period before the first payment:



$$PV = \frac{P}{i} + \frac{Q}{i^2}$$

If the valuation date is not one period before the first payment, then multiply this expression by the appropriate accumulation or discount factor to get to the valuation date.

Finally, there is one more formula that is worth committing to memory; namely, the present value of a rainbow annuity due. We have



$$PV = (\ddot{a}_{\overline{n}|})^2$$