Module 4

Section 1: Inflation

The following example illustrates how inflation affects the purchasing power of money. Suppose we have \$100 now and a gallon of milk currently costs \$4, but instead of buying 25 gallons of milk today, we invest the \$100 for 2 years at an annual effective rate of interest of $i = 8\%$. After 2 years we have $100(1.08)^2 = 116.64$. However, the price of milk may no longer be \$4. Generally commodity prices will increase over time, and we measure such increases by the **inflation rate**. For example, if the annual inflation rate over the 2-year period is $r = 5\%$, then the cost of a gallon of milk after two years is $4(1.05)^2 = 4.41$. Then at the end of two years, we can purchase $116.64/4.41$ (about 26.45) gallons of milk.

The 8% rate in the previous paragraph is called the **nominal interest rate**. The example illustrates that using a nominal annual interest rate of 8% and an annual inflation rate of 5%, then 25 today is equivalent to about 26.45 in two years. The annual effective interest rate implied by this equivalence is called the **real rate of return** and is denoted i'. For this example, we have

$$25(1 + i')^2 = \frac{116.64}{4.41}$$

Note where all these numeric values came from; i.e. $25 = \frac{\$100}{\$4 \text{ per gallon}}$, $116.64 = \$100(1.08)^2$, and $4.41 = \$4(1.05)^2$ per gallon. Substituting into the above equation, we get

$$\frac{100(1 + i')^2}{4} = \frac{100(1.08)^2}{4(1.05)^2} = \frac{100(1 + i)^2}{4(1 + r)^2}$$

Cancelling off common factors and taking square roots, this example illustrates the main fact relating the nominal annual rate i, the annual inflation rate r, and the annual real rate of return i'; namely,

$$1 + i = (1 + r)(1 + i')$$

Module 4 Section 1 Problems:

1. Using a nominal rate, an initial investment of 1000 accumulates to 1071 after 1 year. Assuming an annual inflation rate of 2%, determine the accumulated value of 1000 after 1 year using the annual effective real rate of return.

2. In order to save for retirement, a 25-year old begins depositing 250 at the beginning of each month, beginning on the 25th birthday. Deposits continue until age 65, with the last deposit one month before the 65th birthday.

   (a) Using a nominal interest rate of 9% compounded monthly, determine the accumulated value on the 65th birthday.

   (b) Assuming the same nominal rate of 9% compounded monthly in part (a), and assuming an inflation rate of 3% compounded monthly, determine the accumulated value on the 65th birthday using the real rate of return.

   (c) Redo part (b) except using an inflation assumption of 4% compounded monthly for the first 20 years and 2% compounded monthly thereafter.

3. The present value of a 30-year annuity immediate with semiannual payments of 1000 is 27,675.56, using an annual real rate of return of i. If the assumed inflation rate is 2% compounded semiannually, determine the implied nominal annual interest rate, compounded semiannually.

Answers to Module 4 Section 1 Problems

1) 1050

2) (a) 1,179,108

(b) 498,329

(c) 545,782

3) 8.06%