Section 4: Term Structure of Interest Rates, et al.

IDEA: Not only are short-term and long-term interest rates generally different at any point in time, but they also change over time. This phenomenon is called the **term structure of interest rates**.

The following table is a hypothetical table illustrating the term structure of interest rates. We can extend the values in the table to a continuous graph, and the resulting graphical illustration is called the **yield curve** corresponding to the table. The interest rates in the table are called **spot rates**; these are today's rates.

Hypothetical Term Structure of Interest Rates

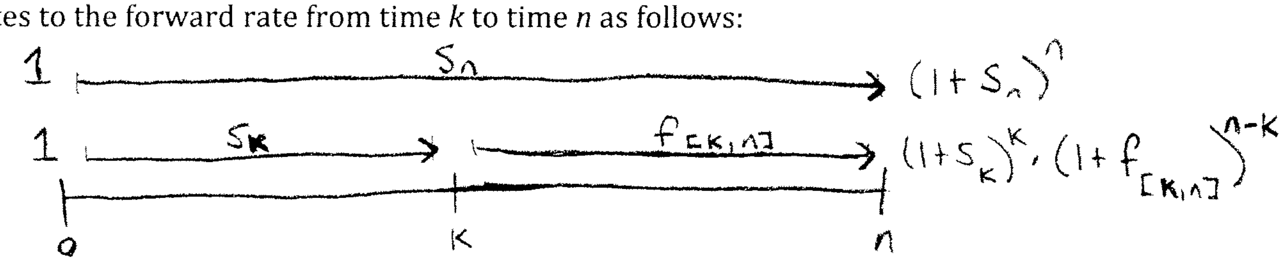
Length of Investment	Interest Rate
1 year	7.00%
2 years	8.00%
3 years	8.75%
4 years	9.25%
5 years	9.50%

The notation we use for spot rates is s_1 for the 1-year spot rate (7% in the table above), s_2 for the 2-year spot rate (8% above), etc. The following example illustrates how we use spot rates from yield curves.

If person A invests 100 for 2 years and person B invests 100 for 3 years, both using the corresponding spot rates in the table above, then A would have $100(1.08)^2 = 116.64$ at the end of 2 years, whereas B would have $100(1.0875)^3 = 128.61$ at the end of 3 years.

Since both A and B started with the same amount, a natural question to ask is "What annual effective interest rate must A receive between year 2 and year 3 so that A has the same amount at the end of year 3 as B does?" It should be clear that the initial amount of 100 is irrelevant, and we solve $(1.0875)^3 = (1.08)^2(1+i)$. This i is called the **forward rate** from time 2 to time 3, and I'll denote it by $f_{[2.3\}}$.

Unless told otherwise, all the spot rates and forward rates are annual effective. Similar to the last paragraph, if $k < n$ then we can relate the k-year and n-year spot rates to the forward rate from time k to time n as follows:



$$(1+s_n)^n = (1+s_k)^k(1+f_{[k,n]})^{n-k}$$

Module 4 Section 4 Problems:

1. Given a two-year spot rate of 4% and a five-year spot rate of 5%, determine the annual forward rate from time 2 to time 5.

2. Suppose the current 1-year spot rate is 3% and the forward rate from time 1 to time 2 consistent with the current term structure of interest rates is 2%. Determine the 2-year discount factor from time 2 back to time 0.

3. You are given 1-year and 2-year spot rates of 4% and 5%, respectively, and a forward rate from time 2 to time 3 of 8%.

   (a) Determine the price of a 1000 face value 3-year bond with 3% annual coupons, redeemable at par, that is consistent with this term structure of interest rates.

   (b) Determine the corresponding annual yield that is consistent with this term structure of interest rates for 3-year 3% annual coupon bonds. [Note: The face value is not given, and in fact, we will show that the annual yield is independent of the face value. Therefore you may use any face value you want. Also since we cannot proceed otherwise, assume the bond is redeemable at par.]

4. The annual yield rate on zero coupon bonds with duration k years is given by

$$i_k = 0.03 + 0.005k, \qquad k = 1, 2, 3$$

   Determine the annual forward rate from time 1 to time 3 that is consistent with these yields.

   Note: A zero-coupon bond is just that; a bond that has no coupons and only pays the redemption value at maturity. For a k-year zero-coupon bond, an investor pays the price for the bond today in return for the redemption value in k years. Therefore, the yield rate on a k-year zero-coupon is precisely the k-year spot rate.

Answers to Module 4 Section 4 Problems

1) 5.6712%

2) $\frac{1}{1.0506}$

3) (a) 921.09

   (b) 5.95%

4) 0.05