Section 6: Asset-Liability Management (Immunization and Dedication)

Asset-liability management refers to the process of investing money (assets) and using the return on the investment to pay future obligations (liabilities). Since we're dealing with both assets and liabilities, we have a present value function for assets, $P^A(i) = \sum A_t \cdot v^t$, and a present value function for liabilities, $P^L(i) = \sum L_t \cdot v^t$, where A_t and L_t are the amounts of the asset and liability at time t. The **net present value function** is $P(i) = P^A(i) - P^L(i) = \sum R_t \cdot v^t$ where $R_t = A_t - L_t$ is the net value of the payment at time t ("assets – liabilities"). We study 2 methods of asset-liability management.

Method 1: (**Redington and Full Immunization**)

We perform immunization at a certain interest rate, i_0. The idea behind **Redington Immunization**, or just **Immunization**, is to structure assets in such a way that the net present value function has a local minimum of 0 at i_0. Recall from calculus that this means three things:

1. $P(i_0) = 0$,
2. $P'(i_0) = 0$, and
3. $P''(i_0) > 0$.

Remarks about these three statements:

1. From the definition of the net present value function, the first statement says the present value of the assets equals the present value of the liabilities at interest rate i_0.
2. Likewise the second statement implies that the derivative of the present value function of the assets equals the derivative of the present value function of the liabilities at interest rate i_0. Recalling the definition of modified duration, $ModD = -\frac{P'(i)}{P(i)}$, statements 1 and 2 taken together imply that the modified duration of the assets equals the modified duration of the liabilities at interest rate i_0. However, since $ModD = v \cdot MacD$, the same can be said about the duration; i.e. the duration of the assets equals the duration of the liabilities at interest rate i_0.
3. It makes sense to introduce terminology regarding the second derivative of the function $P(i_0)$. Similar to the definition of modified duration (volatility), we define the **convexity**, C, of a sequence of future payments as

   $$C = \frac{P''(i)}{P(i)}$$

   Then statements 1 and 3 taken together imply that the convexity of the assets is *greater than* the convexity of the liabilities.

By the definition of immunization at the interest rate i_0, the net present value function (pv of assets – pv of liabilities) has a local minimum of 0 at i_0. Therefore, if we evaluate the net present value function at another interest rate very close to, but not equal to i_0, then we have a positive net present value; i.e. the present value of the assets is more than the present value of the liabilities. This is true for any small change in the interest rate, in either direction away from i_0. Wow, this is nice, but ...

In reality, there may be no way to achieve immunization. In theory, if we assume a **flat yield curve**, meaning all the spot rates are equal, then it is possible to achieve immunization. In fact, if for each of the liabilities, we invest in an asset that has two payouts, one before the liability is due and one after the liability is due, then we will have created a net present value function that not only has a local minimum of 0 at i_0, but has a *global* minimum of 0 at i_0. Such an arrangement of assets versus liabilities creates what is called **full immunization**. So full immunization at i_0 is a technique to structure assets versus liabilities in a manner that would eliminate the risk of adverse effects created by *all* changes in interest rates away from i_0, whereas (Redington) immunization at i_0 would eliminate the risk of adverse effects created by *small* changes in interest rates away from i_0.

Method 2: Absolute or Exact Matching (also called Dedication)

The idea here is that for given liabilities, invest in assets in such a way that when a liability is due, then the assets return an amount exactly equal to the amount of the liability. Then the net value of the payment at each time is 0, i.e. $R_t = 0$, and so the net present value function is identically zero, i.e. $P(i) \equiv 0$, for any interest rate *i*. Conditions 1 and 2 of immunization are satisfied, but note that condition 3 is *not* satisfied since in this case both the convexity of the assets and liabilities is 0. This method is not an immunization strategy.

Module 4 Section 6 Problems:

1. Liabilities at times 3 and 5 of amounts 3000 and 1000, respectively, are to be immunized with 2-year and 8-year zero coupon bonds. Assume a flat yield curve of 3%.

   (a) Determine the cost to immunize the liabilities.

   (b) Determine the asset amounts at time 2 and at time 8 that are needed in order for the first two conditions of immunization to be satisfied.

   (c) If the zero coupon bonds are 1000 face value bonds, redeemable at par, then how many of each type of bond should be purchased in order to immunize the liabilities. (Assume we can buy any number (even fractions) of bonds.)

   (d) Given immunization as above, determine the excess of the present value of assets over the present value of liabilities if the interest rate is changed to 6% annual effective, and then determine the excess of the present value of assets over the present value of liabilities if the interest rate is changed to 1% annual effective.

2. Liabilities of 10,000 in one year and 15,000 in three years are to be exactly matched using 1-year and 3-year zero coupon bonds. Determine the cost to do so if the current 1-year spot rate is 3% and the forward rate from time 1 to time 3 that is consistent with the current term structure of interest rates is 3.5%.

3. A 2-year loan has a required payment of 14,420 in one year and a required payment of 15,900 in two years. These payments are to be exactly matched by investing in 100 face-value 1-year bonds with a 4% annual coupon, and 10,000 face-value 2-year bonds with 6% annual coupons. All bonds are redeemable at par.

   (a) Determine the number of each type of bond that is needed.

   (b) Determine the price to exactly match the payments if both bonds can be bought to yield 4% annual effective.

   (c) Determine the price to exactly match the payments if the 1-year bond can be bought to yield 4% annual effective and the 2-year bond can be bought to yield 5% annual effective.

Answers to Module 4 Section 6 Problems

1) (a) 3608.03

(b) 2884.76 @ $t=2$
1126 @ $t=8$

(c) 2.88476 of the 2-year bond
1.126 of the 8-year bond

(d) 7.78 if $i=.06$
4.52 if $i=.01$

2) 23,303.55

3) (a) 130 of the 1-year bond
1.5 of the 2-year bond

(b) 28565.83

(c) 28278.91