4/4/19

12/midterm) Q: \[ E[Y^p] \]

\[ X \sim \text{Exp} (\theta = 1000) \quad d = 200 \text{ franchise deductible} \]
\[ u = 2000 \text{ policy limit} \]

\[ Y^L = \begin{cases} 
0 & \text{if } x < 200 \\
x & \text{if } 200 \leq x < 2000 \\
2000 & \text{if } x \geq 2000 
\end{cases} \]

\[ Y^P = Y^L \mid X > 200 \]

\[ \therefore E[Y^P] \text{ is an expected value of a conditional random variable.} \]

General situation:

\[ E[g(X) \mid X > a] = \int_a^\infty g(x) \cdot f_x(x) \, dx \]

\[ f_x(x) = \begin{cases} 
0 & \text{if } x \leq a \\
\frac{f_x(x)}{Pr(X > a)} & \text{if } x > a 
\end{cases} \]

\[ \therefore E[g(X) \mid X > a] = \int_a^\infty g(x) \cdot \frac{f_x(x)}{Pr(X > a)} \, dx \]

\[ \therefore \#12 \quad E[Y^P] = \int_{200}^{2000} x \cdot \frac{f(x)}{Pr(X > 200)} \, dx + \int_{2000}^{\infty} 2000 \cdot \frac{f(x)}{Pr(X > 200)} \, dx \]
Remark: \[
\frac{\int_{-\infty}^{\infty} x f(x) \, dx + \int_{200}^{\infty} f(x) \, dx}{\Pr(X > 200)} = \frac{E[Y^2]}{\Pr(X > 200)}
\]

### M453: Estimating Parameters

#### Set-up:
- Given a random variable \( X \) that has \( k \) parameters, we'll have a sample of size \( n \).

\[X: x_1, x_2, \ldots, x_n\]

#### Method 1: Method of Moments (Moment-Matching)

- First, find the empirical distribution, \( \hat{X} \), from the sample.

**Example**: \( X: 15, 20, 15, 30 \)

Then, the empirical distribution for this sample is:

<table>
<thead>
<tr>
<th>( \hat{X} )</th>
<th>( \Pr )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2/4 = 0.5</td>
</tr>
<tr>
<td>20</td>
<td>0.25</td>
</tr>
<tr>
<td>30</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Then match the 1\(^{st}\) \( k \) moments of \( X \) to the 1\(^{st}\) \( k \) moments of \( \hat{X} \)

i.e.: \( E[X^1] = E[\hat{X}^1] \)
\( E[X^2] = E[\hat{X}^2] \)
\( \vdots \)
Method 2: Maximum likelihood Estimation MLE

Idea: Choose the parameters to maximize the "probability" that what happened happens.

Example: \( N \sim \text{Binomial}(m=10, q) \)

\( N: 4, 2, 7 \)

Q: Find the MLE of \( q \) \( (0 \leq q \leq 1) \)

A: For this sample, the likelihood function is

\[
L(q) = \Pr(N=4) \cdot \Pr(N=2) \cdot \Pr(N=7)
\]

\[
= \left[ \binom{10}{4} q^4 (1-q)^6 \right] \cdot \left[ \binom{10}{2} q^2 (1-q)^8 \right] \cdot \left[ \binom{10}{7} q^7 (1-q)^3 \right]
\]

Fact: The location of the maximum value of \( L(q) \) is the same as the location of the maximum value of the expression defining \( L(q) \), ignoring the constant factors.

\[
L(q) \propto q^4 \cdot (1-q)^6\quad \text{definition} \quad \hat{L}(q)
\]

We seek the location of the maximum value of \( \hat{L}(q) \)

Fact: The location of the maximum value of an expression is the same as the location of the maximum value of the natural log of the expression.
Notation: \( L(q) = \text{likelihood function} \)
\( \tilde{L}(q) = \ln(L(q)) = \text{loglikelihood function} \)

Maximize \( \tilde{L}(q) = \ln(\tilde{L}(q)) = 13 \ln(q) + 17 \ln(1-q) \)

Set \( \tilde{L}'(q) = 0 \) and solve