Show all work for full credit, use correct notation, and clearly mark your answer.

1. For a stop-loss insurance on a group of three independent people, you are given:

   (i) Loss amounts are independent

   (ii) The distribution of loss amount for each person is:

<table>
<thead>
<tr>
<th>Loss Amount</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

   \[ E[X] = .3 + .4 + .3 = 1 \]

   (iii) The stop-loss insurance has a deductible of 1.5 for the group.

   Calculate the net stop-loss premium, \( E[(S - 1.5)_+] \).

   \[
   E[(S - 1.5)_+] = E[S] - E[S \wedge 1.5]
   \]

   \[ E[S] = 3 \]

   \[
   \begin{array}{c|c|c}
   S \wedge 1.5 & P_r & Pr \\
   \hline
   0 & (0.4)^3 & 0.064 \\
   1 & 3(0.3)(0.4)^2 & 0.144 \\
   1.5 & 0.792 & 0.792 \\
   \end{array}
   \]

   \[
   \therefore E[S \wedge 1.5] = 1.332
   \]

   \[ E[(S - 1.5)_+] = 3 - 1.332 = 1.668 \]
2. The number of claims in a period has a geometric distribution with mean 3. The amount of each claim follows $Pr(X = x) = 0.25$, $x = 1,2,3,4$. The number of claims and the claim amounts are independent.

Calculate the probability that the aggregate claim amount in the period equals 4.

$$g_4 = p_1 \cdot f_4 + 2 \cdot p_2 \cdot f_1 \cdot f_3 + 3 \cdot p_3 \cdot f_1^2 \cdot f_2 + p_4 \cdot f_4^4$$

$$p_k = \frac{3^k}{4^{k+1}} \quad f_k = \frac{1}{25}$$

$$= \frac{3}{16} \cdot (0.25) + 2 \cdot \frac{9}{64} \cdot (0.25)^3 + \frac{9}{64} \cdot (0.25)^3$$

$$+ 3 \cdot \frac{27}{256} \cdot (0.25)^3 + \frac{81}{1024} \cdot (0.25)^4 = 0.0785$$

3. You are given:

(i) A sample of losses includes exact values of 600, 800, 800, 900, 900, and two more losses that are known to be greater than 1000

(ii) No information is available about losses that are less than 500

(iii) Losses are assumed to follow an exponential distribution with mean $\theta$

Calculate the maximum likelihood estimate of $\theta$.

$$Y = X - 500 \mid X > 500$$

$$Y! 100 \ 300 \ 300 \ 400 \ 400 \ 500^+ \ 500^+$$

$$Y \sim Exp(\theta)$$

$$MLE \Rightarrow \hat{\theta} = \frac{100 + 2(300) + 2(400) + 2(500^+)}{5} = 500$$
4. You are given:

(i) The distribution of the number of claims per policy during a one-year period for 10,000 insurance policies is characterized as follows:

7,500 policies had 0 claims during the year
2,500 policies had 1 claim during the year

You fit a binomial model with parameters \( m = 2 \) and unknown \( q \).

\[ N \sim B(m = 2, q) \]

Calculate the maximum likelihood estimate of \( q \).

\[ \text{MLE} \Rightarrow E[N] = \theta q = \bar{x} = \frac{2500}{10000} \]

\[ \Rightarrow \hat{q} = \frac{1}{\bar{x}} = 1.25 \]

5. A random sample of three claims from an insurance plan is given below:

\[ 700 \quad 825 \quad 975 \]

Claims are assumed to follow a two-parameter Pareto distribution with parameters \( \theta = 250 \) and unknown \( \alpha \).

Calculate the maximum likelihood estimate of \( \alpha \).

\[ L(\alpha) \propto \frac{\alpha^3 \cdot 250^{3\alpha}}{(700)(825)(975)} \]

\[ \bar{L}(\alpha) \propto 3 \ln(\alpha) + 3 \cdot \ln(250) - \alpha \cdot \ln \left( \frac{250}{700} \right) \]

\[ \bar{L}'(\alpha) = \frac{3}{\alpha} + 3 \cdot \ln(250) - \ln \left( \frac{250}{700} \right) = 0 \]

\[ \Rightarrow \alpha = \frac{3}{\ln(1251.631) - 3 \ln(250)} = 0.6815 \]
6. The number of claims per year, \( N \), follows a geometric distribution with mean, \( \beta \). A random sample of 5 observations produces a sum of 10 claims. Use the delta method to approximate the variance of the maximum likelihood estimator of \( p_0 = \Pr(N = 0) \).

\[
\begin{align*}
    p_0 &= (1+\beta)^{-1} \\
    \hat{p}_0 &= (1+\beta)^{-1} \\
    \hat{p}_0 &= (1+\beta)^{-1} \\
    mLE \Rightarrow \hat{\beta} &= \bar{N} = 2 \\
    mLE \Rightarrow \hat{\beta} &= \bar{N} = 2 \\
    \Rightarrow \Var(\hat{\beta}) &= \Var(\bar{N}) = \frac{\Var(N)}{5} = \frac{\beta(1+\beta)}{5} \\
    \Rightarrow \Var(\hat{\beta}) &= \frac{2 \beta(1+\beta)}{5} = 1.2 \\
    \Var(\hat{p}_0) &\text{ Delta method} \\
    &= \frac{1}{81} \cdot (1.2)
\end{align*}
\]

7. You are given the following random sample of 10 loss amounts:

\[
X: 38 \quad 44 \quad 50 \quad 52 \quad 54 \quad 59 \quad 61 \quad 63 \quad 63
\]

Using the estimator from the empirical distribution, calculate the 90% linear symmetric confidence interval for \( \pi = \Pr(X > 55) \).

\[
\hat{\pi} = \frac{4}{10} \quad \hat{\pi} = \frac{N}{10} \quad \text{where} \quad N \sim B(n=10, \pi)
\]

\[
\Var(\hat{\pi}) = \frac{\Var(N)}{100} = \frac{10 \cdot \pi \cdot (1-\pi)}{100} \approx \frac{4.4 \cdot (1-0.4)}{10} = 0.024
\]

\[
\therefore \ CI: \quad \frac{4}{10} \pm 1.645 \sqrt{0.024} \\
(0.145, 0.655)
\]
8. You are given the following information about the aggregate loss for each of \( n \) independent individuals in an auto liability book of business:

(i) The number of annual losses has a Poisson distribution with mean 0.5

(ii) The size of each loss has an exponential distribution with mean 1000

(iii) The amounts of the losses and the number of losses are mutually independent

(iv) There is an ordinary deductible of 100 per loss

\[ \Pr(X > 100) = e^{-1} \]

Determine the minimum value of \( n \) such that when using the normal approximation, there is a 5% probability that the total aggregate losses for the book of business is greater than 110% of its expected value.

\[ T = \sum_{i=1}^{n} S_i \quad S = \sum_{i=1}^{N^L} (Y_i^L) = \sum_{i=1}^{N^P} (Y_i^P) \]

\[ N^P \sim P(\lambda = 5, e^{-1}) \]

\[ Y^P \sim \text{Exp}(\theta = 1000) \]

\[ \therefore E[T] = n \cdot E[S] = 500n e^{-1} \]

\[ \text{Var}(T) = n \cdot \text{Var}(S) = n \cdot 5e^{-1} \cdot 2 \cdot 1000^2 = 100000n e^{-1} \]

\[ 0.05 = \Pr(T > 1.1E[T]) = \Pr\left(\frac{S_{ND}}{\sqrt{\text{Var}(T)}} > \frac{0.1E[T]}{\sqrt{\text{Var}(T)}}\right) \]

\[ = 1.645 \]

\[ \therefore \frac{500n e^{-1}}{\sqrt{100000n e^{-1}}} = 1.645 \]

\[ \Rightarrow n = 1196 + \]