MISS: Policy Modifications

E.g. Deductibles, Maximum Covered Amount, Policy Limits, Co-Insurance

Review: \[ A \land B = \min(A, B) \]
\[ (A)_{+} = \begin{cases} A & \text{if } A \geq 0 \\ 0 & \text{if } A < 0 \end{cases} \]

R.V.'s: \[ X = "\text{ground up loss}" \text{ r.v. = severity r.v. from before} \]
\[ Y^L = \text{r.v. insurance payment per loss} \]
\[ Y^P = \text{r.v. insurance payment per payment} \]
\[ Y^P = Y^L \mid Y^L > 0 \]

Part 1: Ordinary Deductible (d)
\[ X = X \]
\[ Y^L = (X - d)_{+} \]
\[ Y^P = X - d \mid X > d \]

Examples:

1) \[ d = 500 \]

<table>
<thead>
<tr>
<th>X</th>
<th>P_X</th>
<th>Y^L</th>
<th>P_Y^L</th>
<th>Y^P</th>
<th>P_Y^P</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>.4</td>
<td>0</td>
<td>.4</td>
<td>300</td>
<td>.1/3</td>
</tr>
<tr>
<td>800</td>
<td>.2</td>
<td>300</td>
<td>.2</td>
<td>500</td>
<td>.2/3</td>
</tr>
<tr>
<td>1000</td>
<td>.4</td>
<td>500</td>
<td>.4</td>
<td>800</td>
<td>.4</td>
</tr>
</tbody>
</table>

See below
\[ \Pr(Y^p = 300) = \Pr(X = 800 \mid X > 500) = \frac{\Pr(X = 800 \text{ and } X > 500)}{\Pr(X > 500)} = \frac{\Pr(X = 800)}{\Pr(X = 800) + \Pr(X = 1000)} = \frac{1/2}{3/5} = \frac{5}{6} \]

\[ \Pr(Y^p = 500) = 1 - \frac{5}{6} = \frac{1}{6} \text{ (or use the same process above)} \]

Remark: \[ E[Y^1] = 300 \cdot (1/2) + 500 \cdot (1/4) = 260 \]
\[ E[Y^p] = 300 \cdot (1/3) + 500 \cdot (2/3) = \frac{1300}{3} = 433.\bar{3} \]

Note: \[ \Pr(X > 500) = .6 \text{ (probability of a payment)} \]
\[ (433.\bar{3}) \cdot (.6) = 260 \]

Generally, \[ E[Y^k] = E[(Y^p)^k] \cdot \Pr(X > X) \]
for all \( k \) (and all distributions)

Other Commonly Tested Examples
\[ X \sim \text{Exp}(\Theta) \Rightarrow Y^p \sim \text{Exp}(\Theta) \]
\[ X \sim 2\text{-Pareto}(\alpha, \Theta) \Rightarrow Y^p \sim 2\text{-Pareto}(\alpha', \Theta' = \Theta + d) \]
\[ X \sim U(0, w) \Rightarrow Y^p \sim U(0, w - d) \]

Part 2: Maximum Covered Amount \( (U) \) (No deductible)
\[ X = X \]
\[ Y^p = X \wedge U = Y^p \]
Part 3: Ordinary Deductible (d) & Maximum Covered Amount (u)

\[ X = X \]

\[ y^d = \begin{cases} 
0 & \text{if } X \leq d \\
X - d & \text{if } d < X \leq u \\
u - d & \text{if } X > u 
\end{cases} \]

\[ y^p = y^d \mid X > d \]

Note: \[ y^l = \begin{cases} 
X - X & \text{if } X \leq d \\
X - d & \text{if } d < X \leq u \\
u - d & \text{if } X > u 
\end{cases} \]

\[ y^l = (X \land u) - (X \land d) \]

Remark: The policy limit is the maximum amount paid by the insurer. In this case, the policy limit = u - d

Part 4: Co-Insurance Factor (\( \alpha \))

(No deductible, no maximum covered amount)

\[ X = X \]

\[ y^l = \alpha \cdot X = y^p \]

Part 5: \( \alpha, d, \& u \)

(H.W.) \[ y^l = \{ \} \]
Part 6: Franchise Deductible $(d)$

\[
X = X
\]

\[
Y_L = \begin{cases} 
0 & \text{if } X \leq d \\
X & \text{if } X > d 
\end{cases}
\]

\[
Y^p = X \mid X > d = (X-d) \mid X > d + d \mid X > d
\]

\[
Y^p_{Fr, d} = Y^p_{ord, d} + d
\]