MAP 4934 Name:______________________________

Midterm Date: March 10, 2020

Show all work for full credit, use correct notation, and clearly mark your answer.

1. In order to reward efficiency, a company decides to pay its employees a bonus of 50% times the amount by which the companies expenses are under 400. If company expenses are modeled by a lognormal distribution with $\mu = 5.7$ and $\sigma = 0.75$, determine the expected amount of bonus to be paid to employees.

2. You are given:
   (i) with probability 0.3, X has a Burr distribution with $\alpha = 3, \theta = 100$ and $\gamma = 1$
   (ii) with probability 0.7, X has a Weibull distribution with $\tau = 1$ and $\theta = 100$
   Determine $Var(X)$

3. The distribution of the discrete random variable, N, is such that $p_0 = 0.25$, and $p_k = c\left(1 - \frac{2}{k}\right) p_{k-1}$ for k = 1, 2, … . Determine $Var(N)$

4. X follows a two-parameter Pareto distribution with $\alpha = 2$ and unknown θ. If $E[X - 100|X > 100] = 5E[X]$, determine $E[X - 25|X > 50]$.

5. N^T is a zero-truncated Binomial random variable with m = 10 and q = 0.1. Determine $\Pr(N^T = 2)$.

6. Severity X is an Exponential random variable with mean 1000. An insurance company will pay the amount of each claim in excess of a deductible of 100. Calculate the variance of the payment per loss random variable.

7. The ground-up loss random variable is a two-parameter Pareto distribution with parameters $\alpha = 4$ and $\theta = 1200$. If a franchise deductible of 300 is imposed, determine the expected payment per payment.

8. Given $\Lambda = \lambda$, the (conditional) number of claims, $N|\Lambda = \lambda$, for an insured follows a Poisson distribution with mean λ. Λ follows a Gamma distribution, independent of N. The unconditional number of claims, N, for a random insured follows a distribution with mean 0.10 and variance 0.15. Determine the variance of Λ.

9. X, the severity random variable for losses in 2020, has $f_X(x) = 0.1e^{-0.1x}$ for $x > 0$. Inflation of 10% impacts all losses uniformly for the next year, 2021. In 2021, a deductible of 2 is applied to all losses. Y is the payment random variable per payment made in 2021. Find $F_Y(5)$.

10. The ground-up loss X is a Uniform random variable on the interval [0,1000]. Let $M_3 = \max\{X_1, X_2, X_3\}$ for three mutually independent observations of X. Let $A = \Pr(M_3 > 400)$ and $B = \Pr(X > 400)$. Determine A/B.

11. The ground-up loss X is an Exponential random variable with mean 1000, and there is an ordinary deductible of d = 200 and a coinsurance factor of $\alpha = .8$. Determine the expected payment per loss.

12. The ground-up loss X is an Exponential random variable with mean 1000. There is a franchise deductible of d = 200 and a policy limit of u = 2000. Determine the expected payment per payment.

13. The loss random variable X follows a Weibull distribution with parameters $\theta = 10$ and $\tau = 2$. Y is the payment per payment random variable when a deductible of 4 is established. Find $\Pr\,(Y < 8)$.

14. The numbers N of losses each year are independent and identically distributed as Geometric random variables with mean 2. Find the probability that there is at most one claim during a five-year period.

15. The length of time, in years, that a person will remember an actuarial statistic is modeled by an exponential distribution with mean $1/_{Y}$. In a certain population, Y has a gamma distribution with $\alpha = \theta = 2$. Calculate the probability that a person drawn at random from this population will remember an actuarial statistic less than one year.

16. The severity, X, of a loss follows a normal distribution with mean θ and variance 8000, where θ follows a normal distribution with mean 1000 and variance A. Given $\Pr(X > 1150) = 0.0668$, determine A.

17. You are given the following probabilities from a 0-modified $(a, b, 0)$ distribution.

    $p_1^M = \frac{1}{4}$ $\qquad$ $p_2^M = \frac{3}{16}$ $\qquad$ $p_3^M = \frac{9}{64}$

    Determine p_0^M

18. A claim count distribution can be expressed as a mixed Poisson distribution. The mean of the Poisson distribution is uniformly distributed over the interval [0,10]. Determine the variance of the number of claims.

19. A claim count distribution is a 30% / 70% mixture of a Poisson distribution with mean equal to 3 with a geometric distribution with mean equal to 3. Determine the probability that there will be at least two claims.

20. For each individual in a family of three, the annual number of driving accidents follows a binomial distribution with parameters $m = 2$ and $q = 0.2$. The annual numbers of driving accidents for the family members are mutually independent. Determine the probability of exactly 2 driving accidents in a year for the family.