Using Duration to Approximate Changes in Price (Present Value)

Recall the following formulas from Module 4:

Macaulay Duration:

\[ MacD = \frac{\sum t \cdot R_t \cdot v^t}{\sum R_t \cdot v^t} \]

Modified Duration:

\[ ModD = -\frac{P'(i)}{P(i)} = v \cdot MacD \]

\[ P(i) \] is the present value, or price, as a function of \( i \).

We can use these durations to approximate the change in price (present value) for a given change in interest rates. Namely, if the interest rate changes from \( i_{old} \) to \( i_{new} \), then

1. The **first-order modified approximation** for the change in price is

\[ \Delta P \approx -P \cdot ModD \cdot \Delta i, \text{ where } P \text{ and } ModD \text{ use } i_{old}, \text{ and } \Delta i = i_{new} - i_{old} \text{ is the change in the interest rate.} \]

2. The **first-order Macaulay approximation** for the change in price is

\[ \Delta P \approx P \cdot \left[ \left( \frac{1+i_{old}}{1+i_{new}} \right)^{MacD} - 1 \right], \text{ where } P \text{ and } MacD \text{ use } i_{old}. \]

Example: A bond has a price of 7025 using an annual effective yield rate of 7%.
Using the same yield rate, the Macaulay duration of the bond 4.946 years.

(a) Using the first-order modified approximation, calculate the price of this bond if the yield rate is change to 6.5% annual effective.
(b) Using the first-order Macaulay approximation, calculate the price of this bond if the yield rate is change to 6.5% annual effective.
(c) Using the first-order modified approximation, calculate the price of this bond if the yield rate is change to 7.2% annual effective.
(d) Using the first-order Macaulay approximation, calculate the price of this bond if the yield rate is change to 7.2% annual effective.
Solution:

(a) \[ \Delta P \approx -P \cdot \text{Mod}D \cdot \Delta i, \text{ where } P = 7025, \text{Mod}D = \frac{4.946}{1.07}, \text{ and } \Delta i = -0.005 \]

Therefore, \[ \Delta P \approx -(7025) \cdot \left(\frac{4.946}{1.07}\right) \cdot (-0.005) = 162.362 \ldots, \text{ and so the new price is approximately } P + \Delta P \approx 7025 + 162 = 7187. \]

(b) \[ \Delta P \approx P \cdot \left[ \left(\frac{1+i_{old}}{1+i_{new}}\right)^{\text{Mac}D} - 1 \right], \text{ where } P = 7025 \text{ and Mac}D = 4.946. \]

Therefore, \[ \Delta P \approx (7025) \cdot \left[ \left(\frac{1.07}{1.065}\right)^{4.946} - 1 \right] = 164.643 \ldots, \text{ and so the new price is approximately } P + \Delta P \approx 7025 + 165 = 7190. \]

(c) \[ \Delta P \approx -P \cdot \text{Mod}D \cdot \Delta i, \text{ where } P = 7025, \text{Mod}D = \frac{4.946}{1.07}, \text{ and } \Delta i = 0.002 \]

Therefore, \[ \Delta P \approx -(7025) \cdot \left(\frac{4.946}{1.07}\right) \cdot (0.002) = -64.945 \ldots, \text{ and so the new price is approximately } P + \Delta P \approx 7025 - 65 = 6960. \]

(d) \[ \Delta P \approx P \cdot \left[ \left(\frac{1+i_{old}}{1+i_{new}}\right)^{\text{Mac}D} - 1 \right], \text{ where } P = 7025 \text{ and Mac}D = 4.946. \]

Therefore, \[ \Delta P \approx (7025) \cdot \left[ \left(\frac{1.07}{1.072}\right)^{4.946} - 1 \right] = -64.585 \ldots, \text{ and so the new price is approximately } P + \Delta P \approx 7025 - 65 = 6960. \]