MIS1: Accumulation Function

\[ a(t) = paf_0^t \]

\[ Y = X \cdot paf_k^n = X \cdot \frac{a(n)}{a(k)} \]

\[ X = Y \cdot pdf_n = Y \cdot \frac{a(k)}{a(n)} \]

MIS2: Simple & Compound Interest

\[ i = \text{simple interest} \implies a(t) = 1 + i \cdot t \quad (t \text{ in years}) \]

\[ i = \text{periodic effective interest rate} \implies a(t) = (1+i)^t \quad (t \text{ of periods}) \]

Examples:

1) \( i = 0.06 \) simple \implies a(t) = 1 + 0.06t

\[ pdf_{4}^1 = \frac{a(1)}{a(4)} = \frac{1.06}{1.24} = 0.8548 \ldots \]

\[ pdf_{8}^5 = \frac{a(5)}{a(8)} = \frac{1.2}{1.48} = 0.8782 \ldots \]

2) \( i = 0.06 \) annual effective interest rate (AEIR)

\[ a(t) = 1.06^t \quad (t \text{ in years}) \]

\[ pdf_{4}^1 = \frac{a(1)}{a(4)} = \frac{1.06}{1.06^4} = (1.06)^{-3} = 0.8396 \ldots \]

\[ pdf_{8}^5 = \frac{a(5)}{a(8)} = \frac{1.06^5}{1.06^8} = (1.06)^{-3} = 0.8396 \ldots \]

"\( V \)-notation" \( V = \text{the periodic discount factor} \)

\[ = \frac{1}{1+i} \quad i = \text{periodic AEIR} \]
In this example \( c = 0.06 = acfr \)

\[ \implies a = \frac{1}{1.06} = \text{a d f} \]

Note: \( v^1 = 1.06 = \text{a a f} \)

\[ v^2 = \left(\frac{1}{1.06}\right)^2 = \text{ba a d f} \]

\[ v^{1/2} = \text{sa a f} = (1.06)^{1/2} \]

**MIS3: Simple & Compound Discount**

- Simple discount rate \( \implies a(t) = (1 - dt)^{-1} \) (\( t \)-in years)

- Periodic effective discount rate

\[ \implies a(t) = (1 - d)^{-t} \]

**Example:** \( d = 0.06 \) simple \( \implies a(t) = (1 - 0.06t)^{-1} \)

\[ p d f_{10}^4 = \frac{a(4)}{a(10)} = \frac{0.76^{-1}}{0.4^{-1}} = \frac{0.4}{0.76} = 0.5263 \ldots \]

\[ p d f_5 = \frac{a(1)}{a(5)} = \frac{0.94^{-1}}{0.7^{-1}} = \frac{0.7}{0.94} = 0.7446 \ldots \]

\[ d = 0.06 \text{ aedr} \implies a(t) = (0.94)^{-t} \]

\[ p d f_{k+1}^k = \frac{a(k)}{a(k+1)} = \frac{(0.94)^{-k}}{(0.94)^{-k+1}} = \frac{0.94^{k+1}}{0.94^k} = 0.94 \]

\[ v = 0.94 = \text{a d f} = 1 - \text{a e d r} \]
When compounding:

\[ i = \text{peir} \quad \text{periodic effective interest rate} \]

\[ d = \text{pedr} \quad \text{discount rate} \]

\[ \text{paf} \quad \text{periodic accumulation factor} \]

\[ \text{pdf} \quad \text{periodic discount factor} \]

\[ S' = \text{paf} = 1 + i = (1 - d)^{-1} \]

\[ z = \text{pdf} = 1 - d = (1 + i)^{-1} \]