Accumulation Functions (Summary)

1) i-simple $\Rightarrow a(t) = (1 + i \cdot t)$ \hspace{1cm} t in years

2) i-geir $\Rightarrow a(t) = (1 + i)^t$ \hspace{1cm} t - # of periods

3) d-simple $\Rightarrow a(t) = (1 - dt)^{-1}$ \hspace{1cm} t in years

4) d-geir $\Rightarrow a(t) = (1 - d)^{-t}$ \hspace{1cm} t - # of periods

5) $S_t$ - foci $\Rightarrow a(t) = \int_0^t S_r \, dr$

\hspace{1cm} (a) $S_t = S$ $\Rightarrow a(t) = e^{St}$

\hspace{1cm} (b) $S_t = C \cdot \frac{f'(t)}{f(t)}$ $\Rightarrow a(t) = \left[\frac{f(t)}{f(0)}\right]^C$

Examples (See next pages)
An account credits interest using a simple discount rate, $d$. A deposit of $X$ at time $t = 0$ accumulates to $2X$ at time $t = 5$. Determine the time at which the account will have $4X$.

(A) 7.5

$$a(t) = (1 - dt)^{-1}$$

(B) 8.0

$$X \cdot a(5) = 2X \implies a(5) = 2 = (1 - 5d)^{-1}$$

(C) 8.5

$$\implies d = 0.1$$

(D) 9.0

$$X \cdot a(n) = 4X \implies a(n) = 4 = (1 - 10n)^{-1}$$

(E) 10.0

$$\implies n = 7.5 \quad \text{(A)}$$
An account credits interest using a simple interest rate, \( i \), for the first three months, then a discount rate of 6%, convertible monthly, for the next nine months. Thereafter, the account credits interest using an interest rate of \( i \), payable quarterly, which is equivalent to an annual effective discount rate, \( d \). An initial deposit of $8071 accumulates to $10,000 at the end of five years. Determine \( d \).

(A) 3.8%

(B) 3.9%

(C) 4.0%

(D) 4.1%

(E) 4.2%

\[
0.06 = d^{(12)}
\]

\[
10000 = \frac{8071}{\left(1 + \frac{i}{4}\right) \cdot \left(1 - \frac{0.06}{12}\right) \cdot \left(1 + \frac{i}{4}\right)^{16}}
\]

\[
\Rightarrow i = \left[\frac{10000 \cdot 0.995^9}{8071}\right]^{\frac{1}{15}} - 1 \Rightarrow i = 0.041 \ldots
\]

\[
0.04 = d^{(4)} \Rightarrow \ae dr = d = ?
\]

\[
\ae f = \left(1 + \frac{i}{4}\right)^4 = \left(1 - d\right)^{-1} \Rightarrow d = 0.039 \ldots
\]