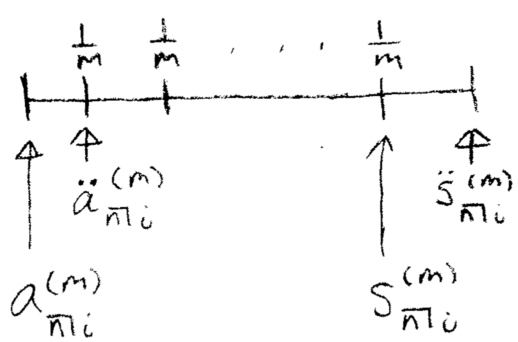
Section 3: Basic upper m Notation

We have already seen “upper m” notation in the context of nominal interest rates. In the context of annuities, we use upper m notation when the annuity payments are more frequent than interest is compounded. The last two problems in the previous problem set illustrate such problems. We can see from the solutions of these problems that we can generally determine annuity values for such annuities by converting the given interest rate to its equivalent periodic effective interest rate, where the period matches the payment period. When possible, this is the approach most students take in such problems. However, you may see (a very few) problems on your actuarial exam that make use of the following upper m annuity notation.

Without loss of generality, we assume an n year annuity with payments made m times per year, for a total of nm payments, and we assume we’re given an annual effective interest rate i. Just as with regular annuities, the term “basic” will represent the case that the annual payment is 1. Since there are m payments per year, then each payment is $1/m$. The timeline for “m^{thly} annuities” is:



period $= \left(\frac{1}{m}\right)^{\text{th}}$ of a year

$(n \cdot m)$ payments of $\frac{1}{m}$

i = aeir

n = # of years

Formulas for “m^{thly} annuities”:

$$a^{(m)}_{\overline{n}|i} \overset{VEP}{=} \frac{1}{m}v^{\frac{1}{m}} + \frac{1}{m}v^{\frac{2}{m}} + \cdots + \frac{1}{m}v^{n} \overset{CRF}{=} \frac{1-v^n}{i^{(m)}}$$

$$\ddot{a}^{(m)}_{\overline{n}|i} \overset{VEP}{=} \frac{1}{m} + \frac{1}{m}v^{\frac{1}{m}} + \frac{1}{m}v^{\frac{2}{m}} + \cdots + \frac{1}{m}v^{\frac{nm-1}{m}} \overset{CRF}{=} \frac{1-v^n}{d^{(m)}}$$

$$s^{(m)}_{\overline{n}|i} \overset{VEP}{=} \frac{1}{m} + \frac{1}{m}(1+i)^{\frac{1}{m}} + \frac{1}{m}(1+i)^{\frac{2}{m}} + \cdots + \frac{1}{m}(1+i)^{\frac{nm-1}{m}} \overset{CRF}{=} \frac{(1+i)^n - 1}{i^{(m)}}$$

$$\ddot{s}^{(m)}_{\overline{n}|i} \overset{VEP}{=} \frac{1}{m}(1+i)^{\frac{1}{m}} + \frac{1}{m}(1+i)^{\frac{2}{m}} + \cdots + \frac{1}{m}(1+i)^{n} \overset{CRF}{=} \frac{(1+i)^n - 1}{d^{(m)}}$$

$$a^{(m)}_{\overline{\infty}|i} \overset{VEP}{=} \frac{1}{m}v^{\frac{1}{m}} + \frac{1}{m}v^{\frac{2}{m}} + \cdots \overset{CRF}{=} \frac{1}{i^{(m)}}$$

$$\ddot{a}^{(m)}_{\overline{\infty}|i} \overset{VEP}{=} \frac{1}{m} + \frac{1}{m}v^{\frac{1}{m}} + \frac{1}{m}v^{\frac{2}{m}} + \cdots \overset{CRF}{=} \frac{1}{d^{(m)}}$$

Module 2 Section 3 Problems:

1. Determine the present value of a 10-year annuity immediate with monthly payments of 30 using an annual effective interest rate of 6%. (This is Number 11 of the previous problem set.)

2. Determine the accumulated value of a 10-year annuity immediate with monthly payments of 30 using an annual effective interest rate of 6%.

3. Determine the accumulated value of a 15-year annuity due with quarterly payments of 20 using an annual effective interest rate of 3%.

4. Determine the present value of a perpetuity immediate with semiannual payments of 60 using an annual effective interest rate of 6.09%.

5. Determine the present value of a perpetuity due with semiannual payments of 60 using an annual effective interest rate of 6.09%.

6. Determine the present value of a perpetuity due with quarterly payments of 60 using a nominal interest rate of 12.18% compounded semiannually. (This is Number 12 of the previous problem set.)

Answers to Module 2 Section 3 Problems

1) 2721.729649

2) 4874.203273

3) 1515.708403

4) 2000

5) 2060

6) 2060