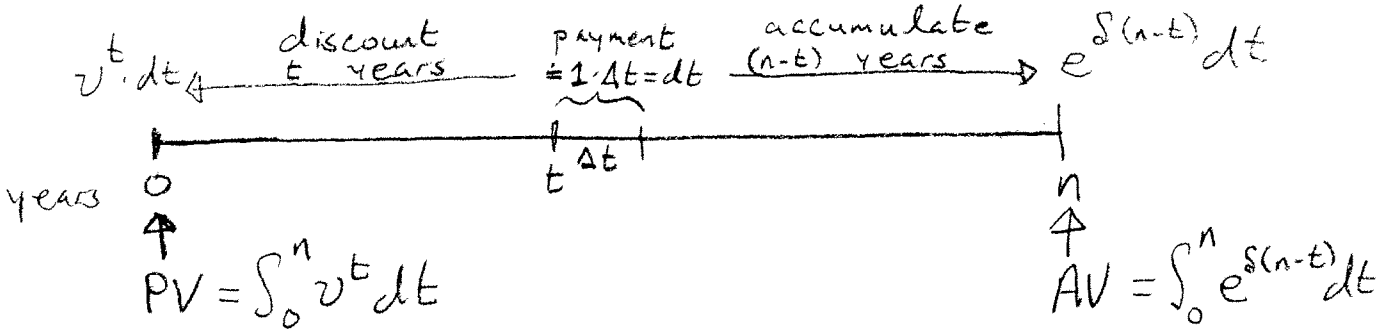
Section 4: Basic Level Continuous Annuities

By continuous annuity, we mean an annuity in which the payments are made continuously throughout the payment period. Of course this is not practical, since payments cannot be made continuously, but there is theoretical value to these annuities. The payment period will always be in years, and the "basic level" part of the title of this section means the total amount paid per year is 1. In this context of continuous annuities, we say the payment rate is 1 per year.

The idea is to start with the m^{thly} annuities from the last section, those for which payments of $1/m$ are made m times per year, and then take the limit as m goes to infinity. The result from this limiting process is called a continuous annuity.

A timeline and formulas to value continuous annuities follow. The VEP formulas can be related back to the VEP formulas from the last section. For example, the VEP formula for the present value of a basic level continuous annuity uses the fact that the VEP's for $\ddot{a}^{(m)}_{\overline{n}|i}$ and $a^{(m)}_{\overline{n}|i}$ are upper and lower Riemann sums for $\int_0^n v^t\,dt$, respectively. We derive the CRF's by integrating in the VEP formulas. It is common to be given a constant force of interest, δ, in continuous annuity problems.

Timeline: (payment rate = 1 per year)



$$\bar{a}_{\overline{n}|} \overset{VEP}{=} \int_0^n v^t\,dt \overset{CRF}{=} \frac{1-v^n}{\delta}$$

$$\bar{s}_{\overline{n}|} \overset{VEP}{=} \int_0^n e^{\delta(n-t)}\,dt \overset{CRF}{=} \frac{e^{\delta n}-1}{\delta}$$

Note that, similar to regular annuities, $\bar{s}_{\overline{n}|} = \bar{a}_{\overline{n}|} \cdot (1+i)^n = \bar{a}_{\overline{n}|} \cdot e^{\delta n}$.

Module 2 Section 4 Problems:

1. Determine the present value of a 10-year annuity with payments made continuously at a rate of 1 per year, using an annual effective interest rate of 6%.

2. Determine the accumulated value of a 20-year annuity with payments made continuously at a rate of 5 per year, using a force of interest equal to 4%.

3. An n-year annuity with payments made continuously at a rate of K per year has a present value of 10000 using an interest rate of 5% per annum. Using the same interest rate, the accumulated value of the annuity is 11025. Determine K.

Answers to Module 2 Section 4 Problems

1) 7.578745463

2) 153.1926161

3) 5247.917658