Section 5: Bond Amortization Schedule

Notation:

We use the same symbols in a bond amortization table as we do in a loan amortization table. In fact, the same relationships hold in a bond amortization table as in a loan amortization table, with the only differences being in terminology. We have

B_k – book value (or amortized value) immediately after the k^{th} coupon payment (this is like the loan balance)

I_k – Amount of the k^{th} coupon that represents the amount of interest earned (instead of the amount or interest paid)

P_k – Amount of the k^{th} coupon that represents principal adjustment (instead of principal repaid)

Basic Relationships for Bond Amortizations:

$B_0 = P = Fra_{\overline{n|}i} + Cv^n$ = the price of the bond
$B_n = C$ = the redemption value of the bond.
(Think of the redemption value as being paid one second after time n.)

$B_k \overset{Pro}{=} Fra_{\overline{n-k|}i} + Cv^{n-k}$ (= PV of future coupons and redemption value, using i)
(We'll see a retrospective formula later.)

Just as with loans, $I_k = i \cdot B_{k-1}$ and then $P_k = Fr - I_k$

These relationships are captured in a **Bond Amortization Table**:

Time	Coupon	Interest Earned	Principal Adjustment	Book Value (Amortized Value)
0				$P = B_0 = Fra_{\overline{n|}} + Cv^n$
1	Fr	$I_1 = i \cdot P$	$P_1 = Fr - I_1$	$B_1 = Fra_{\overline{n-1|}} + Cv^{n-1}$
2	Fr	$I_2 = i \cdot B_1$	$P_2 = Fr - I_2$	$B_2 = Fra_{\overline{n-2|}} + Cv^{n-2}$
⋮	⋮	⋮	⋮	⋮
n	Fr	$I_n = i \cdot B_{n-1}$	$P_n = Fr - I_n$	$B_n = C$

Remarks about this table:

1. $\{P_1, P_2, \cdots, P_n\}$ is a geometric sequence with common ratio $r = 1 + i$.

2. We can relate the book value at time k to the book value at time m ($k < m$) as:

   $B_k = Fra_{\overline{m-k}|} + B_m v^{m-k}$ (Note that this is a one-step TVM calculation.)

   As written, this equation has a valuation date at time k. Multiplying both sides by $(1+i)^{m-k}$ and rearranging terms gives the time m equation $B_m = B_k(1+i)^{m-k} - Rs_{\overline{m-k}|}$. With $k = 0$, this is the prospective method of determining the book value.

3. As a special case of the previous remark, we can calculate book values at neighboring times in two ways:

   $$B_{k+1} = B_k(1+i) - Fr$$

   or

   $$B_{k+1} = B_k - P_{k+1}$$

   The above remarks are identical to the remarks made about a level payment loan amortization table. The next remarks are specific to a bond amortization.

4. Recall that the bond is bought at a premium if $P > C$. In this case, since $B_0 = P$ and $B_n = C$, the book values are systematically *decreasing* from time 0 to time n. We say the bond is **written down** and each of the P_k values is a positive value that we call the **amortization of premium**, or **amount of write-down**, for period (or installment) k.

5. Recall that the bond is bought at a discount if $P < C$. In this case, since $B_0 = P$ and $B_n = C$, the book values are systematically *increasing* from time 0 to time n. We say the bond is **written up** and each of the P_k values is a *negative* value. Exams may refer to the absolute value of the P_k values as the **accumulation of discount**, or **amount of write-up**, for period (or installment) k. Note that accumulation of discount or amount of write-up implies the bond was bought at a discount, and so the P_k's are *negative.*

6. Regardless of whether the bond is bought at a premium or at a discount, $\sum_{i=1}^{n} P_i = P_1 + P_2 + \cdots + P_n = P - C$. Notice that this value is positive if the bond is bought at a premium, but is negative if the bond is bought at a discount.

Module 3 Section 5 Problems:

1. A 10-year 10,000 face value bond with 5% annual coupons is bought to yield 6% compounded annually. Determine the book value immediately after the 4^{th} coupon.

2. A 10-year 10,000 face value bond with 5% annual coupons is bought to yield 6% compounded annually. Determine the book value immediately before the 4^{th} coupon.

3. A bond with semiannual coupons of 5 is bought to yield 4% compounded semiannually. The book value at the end of the 6^{th} year is 162.48. Determine the price paid for the bond.

4. A bond that was bought to yield 3% annual effective has annual coupons of 50. The book value is 1400 immediately after the seventh coupon is paid. Determine the book value immediately after the tenth coupon is paid.

5. A bond for which the book value is 937 immediately after the k^{th} coupon is paid has an accumulation of discount of 8 during period $(k + 1)$. Determine the book value of the bond immediately after the coupon at time $(k + 1)$ is paid.

6. A 10-year 1000 par value bond with 8% annual coupons is bought to yield 6% annual effective. Determine the amount of interest earned during the 3^{rd} year.

7. A bond has a book value, immediately after the 8^{th} coupon is paid, of 878. The coupons are 35 each, and the principal adjustment for the 9^{th} installment is 8.66. Determine the periodic effective yield rate for which the bond was bought.

8. A 20-year 1000 face value bond with 7% semiannual coupons is bought for 901. Determine the amount of interest earned during the 12^{th} year.

9. A 15-year 1000 face value bond with 7% annual coupons and redemption value of 1250 is bought to yield 5% annual effective. Determine the amount of principal adjustment for the eighth year.

10. A 10-year bond with semiannual coupons is bought to yield 8% compounded semiannually. The amortization of premium for the 8^{th} installment is 6.07. Determine the amount of premium for which this bond was bought.

11. A 6-year bond, redeemable at 1500, with quarterly coupons is bought to yield 8% compounded quarterly. The amount of write-up for the 4^{th} installment is 3.57. Determine the price paid for the bond.

Answers to Module 3 Section 5 Problems

1) 9508.27

2) 10008.27

3) 180.99

4) ~~[illegible]~~ 1375.27

5) 945

6) 67.45

7) 3%

8) 75.03

9) 5.08

10) 137.36

11) 1397.66