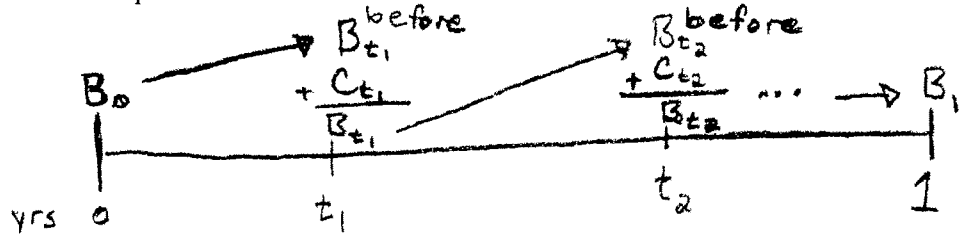
Section 2: Dollar and Time Weighted Rates

Dollar Weighted and Time Weighted Interest Rates are used to evaluate investor performance and investment fund managers' performance.

Set-up: We have a fund account value, or balance, at the beginning of the year, denoted by B_0, a balance at the end of the year, denoted by B_1, and transactions during the year. These transactions during the year, at time t ($0 < t < 1$), are deposits and/or withdrawals denoted by D_t and W_t, respectively. The net contribution for the transaction at time t is $C_t = D_t - W_t$. Of course this may be negative. At each transaction date, there are balances immediately before and immediately after the transaction, that we denote B_t^{before} and B_t, respectively.

We can capture all this information in a timeline as follows:



Dollar-Weighted Return: This return is the constant simple interest rate, i, at which the beginning balance and net contributions are equivalent to the ending balance, using an end of year valuation date. Note that the accumulated value at time $t = 1$ of the beginning balance is $B_0(1 + i)$, and by definition of dollar-weighted return, the accumulated value at time $t = 1$ of the net contribution at time t is $C_t(1 + i(1 - t))$. Therefore, $B_1 = B_0(1 + i) + \sum C_t(1 + i(1 - t))$. Solving this equation for i, we obtain

$$i_{DW} = \frac{B_1 - B_0 - \sum C_t}{B_0 + \sum C_t(1 - t)} = \frac{I}{Exp}$$

where I is the amount of interest earned during the year, and Exp is the *exposure*, which is the amount of the fund "exposed to interest" during the year.

Time-Weighted Return: This return in determined by compounding. We calculate the time-weighted return by setting the annual time-weighted accumulation factor, $1 + i_{tw}$, equal to the product of the periodic accumulation factors, where the periods are determined by the transaction dates. For example, the first periodic accumulation factor is $\frac{B_{t_1}^{before}}{B_0}$, the second periodic accumulation factor is $\frac{B_{t_2}^{before}}{B_{t_1}}$, and so on. Then $1 + i_{tw}$ equals the product of these accumulation factors. The notation makes the calculation of the time-weighted return look complicated, but it's actually easier than the calculation of the dollar-weighted return.

Module 4 Section 2 Problems:

1. At the beginning of the year, an account has a balance of 100,000. There is a deposit of 30,000 on May 1, and there are withdrawals of 6,000 and 27,000 on March 1, and September 1, respectively. There are no other deposits or withdrawals. The account balance at the end of the year is 100,710. Determine the dollar weighted return.

2. Investor A and Investor B each have 1000 in the same investment account on January 1. On April 1, Investor A deposits an additional 900 whereas Investor B withdraws 100. The balance in each account immediately before the transactions is 1100. On October 1, Investor A withdraws 500 and Investor B deposits 750. The balance in Investor A's account immediately before the withdrawal is 2500, whereas the balance in Investor B's account immediately before the deposit is 1250. On December 31, the balance in each account is 1600.

   (a) Determine the dollar-weighted rate of return in Investor A's account.
   (b) Determine the dollar-weighted rate of return in Investor B's account.
   (c) Determine the time-weighted rate of return in Investor A's account.
   (d) Determine the time-weighted rate of return in Investor B's account.

3. An investor has an account with a beginning balance on January 1 of 10,000. The investor makes deposits of 300 at the end of each month. There are no withdrawals from the account. If the account balance is 14473.75 on the following January 1, determine the dollar weighted return the investor received.

4. An account has a beginning of year balance of 5000. On June 20th there is a deposit of 500. There are no other deposits or withdrawals. The end of year balance is 5768 and the time weighted return for the year is 5.06%. Determine the balance in the account on June 20th immediately after the deposit on that date.

5. An account has a dollar weighted return of 10% during the year. The beginning of year balance is 2000. There are no deposits and there is only one withdrawal of 300 during the year. The ending balance is 1876.25. Determine the date of the withdrawal.

Answers to Module 4 Section 2 Problems

1) .035

2) (a) 0.12903

(b) -0.04494

(c) 0.10

(d) 0.10

3) 0.075

4) 5600

5) March 15