MAP 4170 Name:_____

Test 1 Date: September 29, 2011

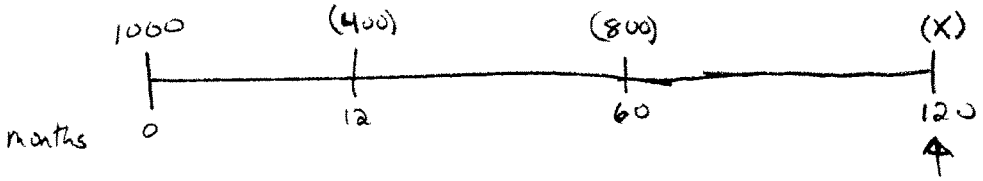
Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan of 1000 is made at an interest rate of 12% compounded monthly. The loan is to be repaid with three payments: 400 at the end of the first year, 800 at the end of the fifth year, and the balance at the end of the tenth year. The present value of the payments equals the loan amount. Determine the amount of the final payment.

$i^{(12)} = 12\%$

$\Rightarrow i = .01 = \text{mo int}$

(A) 613 (B) 675 (C) 737 (D) 799 (E) 861



$$1000(1.01)^{120} - 400(1.01)^{108} - 800(1.01)^{60} - X = 0$$

$$X \doteq 675$$

2. A 10-year loan is originated based on a simple interest rate, i. The periodic effective discount rate on the loan during the 6th year is 4%. Determine i.

(A) 4.25% (B) 4.50% (C) 4.75% (D) 5.00% (E) 5.25%

$$a(t) = 1 + it$$

$$d_6 = \frac{a(6) - a(5)}{a(6)} = \frac{i}{1+6i}$$

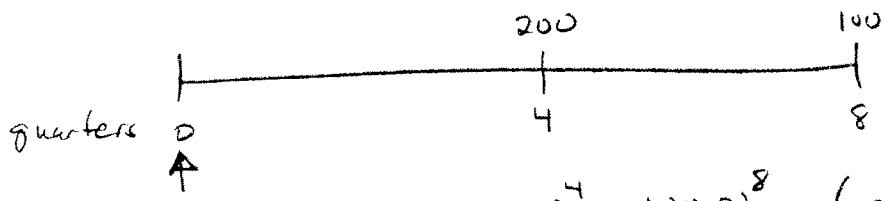
$$\therefore 0.04 = \frac{i}{1+6i} \Longrightarrow i = \frac{4}{76} \doteq 0.0526$$

3. Payments of \$200 at the end of one year, and \$100 at the end of two years, have a present value of \$285 when valued using a quarterly effective interest rate of i. Determine i.

(A) 0.97% (B) 1.71% (C) 2.45% (D) 3.19% (E) 3.93%

$i = \text{qeir}$

$v = \frac{1}{1+i} =$ quarterly discount factor



$PV = 285 = 200v^4 + 100v^8$ (quadratic in v^4)

$a = 100$
$b = 200$
$c = -285$

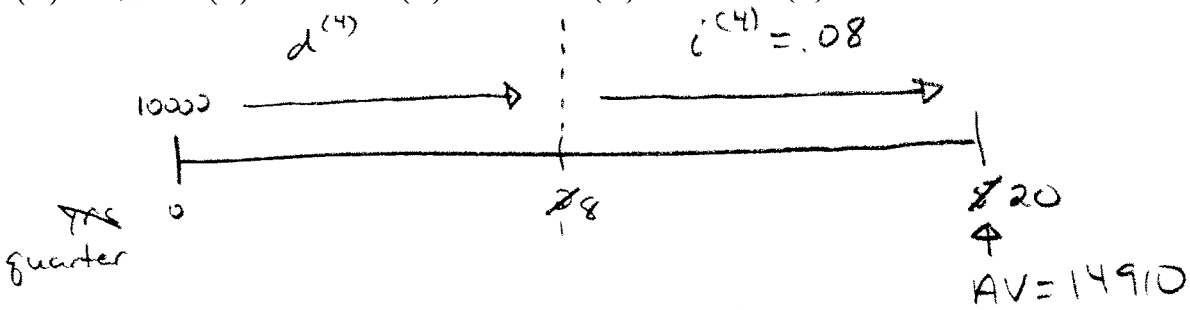
$$v^4 = \frac{-200 \pm \sqrt{200^2 - 4(100)(-285)}}{2(100)} = (1+i)^{-4}$$

$$\Rightarrow i = \left(\frac{-200 \pm \sqrt{200^2 + 4(100)(285)}}{200}\right)^{-1/4} - 1 = 0.97\%$$

4. A deposit of 10,000 is made into a fund at time $t = 0$. The fund pays interest at a nominal rate of discount of d compounded quarterly for the first two years, and thereafter interest is credited using a nominal rate of interest of 8% compounded quarterly. After 5 years, the accumulated value of the fund is 14,910. Calculate d.

(A) 8.0% (B) 8.1% (C) 8.2% (D) 8.3% (E) 8.4%

$d = d^{(4)}$



$$14910 = 10000\left(1 - \frac{d^{(4)}}{4}\right)^{-8}\left(1 + \frac{.08}{4}\right)^{12}$$

$$d^{(4)} \doteq 8.0\%$$

5. You are given $\delta_t = \dfrac{1}{2+2t}$ for $t > 0$. A payment of 400 at the end of 3 years and 800 at the end of 15 years has the same present value as a payment of 300 at the end of 8 years and X at the end of 24 years. Determine X.

(A) 900 (B) 1200 (C) 1500 (D) 1800 (E) 2100

$$PV = \frac{400}{a(3)} + \frac{800}{a(15)} = \frac{300}{a(8)} + \frac{X}{a(24)}$$

$$\delta_t = \frac{1}{2+2t} = \frac{1}{2}\,\frac{2}{2+2t} = c \cdot \frac{f'(t)}{f(t)} \qquad f(t) = 2+2t \quad c = \frac{1}{2}$$

$$\Longrightarrow a(t) = \left[\frac{f(t)}{f(0)}\right]^c = \left(\frac{2+2t}{2}\right)^{1/2} = \sqrt{1+t}$$

$$a(3) = \sqrt{4} = 2 \qquad a(15) = \sqrt{16} = 4 \qquad a(8) = \sqrt{9} = 3 \qquad a(24) = \sqrt{25} = 5$$

$$\therefore \frac{400}{2} + \frac{800}{4} = \frac{300}{3} + \frac{X}{5} \Longrightarrow X = 1500$$

6. Fund A accumulates according to simple discount at a 6% rate. Fund B accumulates according to compound interest at a quarterly effective interest rate of 2%. Let T denote the time, to the nearest tenth of a year, when the force of interest for Fund A equals the force of interest for Fund B.

Determine T.

(A) 3.7 (B) 4.0 (C) 5.6 (D) 12.3 (E) 16.6

A: $a(t) = (1-.06t)^{-1} \Longrightarrow a'(t) = 0.06(1-.06t)^{-2}$

t-yrs

$$\therefore \delta_t^A = \frac{0.06}{1-.06t}$$

B: t must be in years $\qquad i = .02 = \text{qeir}$

Convert to an aeir $= j \qquad 1+j = (1.02)^4$

$$\therefore a(t) = (1+j)^t = 1.02^{4t} \Longrightarrow a'(t) = 1.02^{4t} \cdot \ln(1.02) \cdot 4$$

$$\therefore \delta_t^B = 4 \cdot \ln(1.02)$$

$$\therefore \frac{0.06}{1-.06T} = 4\ln(1.02) \Longrightarrow T \doteq 4.0$$

7. At an annual effective interest rate i, $i > 0$, the following are all equal:

I. The present value of 45,000 at the end of 12 years
II. The sum of the present values of 6000 at the end of year t and 56,000 at the end of year $2t$
III. 5000 immediately

Determine the present value of a payment of 8000 at the end of year $t + 6$ using the same annual effective interest rate.

v = annual discount factor

(A) 100 (B) 250 (C) 333 (D) 500 (E) 667

Determine $8000 \cdot v^{t+6} = 8000 \cdot v^t \cdot v^6$

I. = III.: $45000 v^{12} = 5000 \Rightarrow v^{12} = \frac{1}{9} \Rightarrow v^6 = \frac{1}{3}$

II. = III.: $6000 v^t + 56000 v^{2t} = 5000$ (quadratic in v^t)

$\Rightarrow v^t = 0.25$

$\therefore 8000 v^{t+6} = 8000(0.25)(\frac{1}{3}) \doteq 667$

8. A collection agency pays a doctor \$5,000 for invoices that the doctor has not been able to collect on. After two years, the collection agency has collected \$6,000 on the invoices. Determine the nominal rate of discount compounded monthly the collection agency receive on this transaction over the two year period.

(A) 8.71% (B) 9.08% (C) 9.13% (D) 9.15% (E) 9.55%

Determine $d^{(12)}$

$$5000\left(1 - \frac{d^{(12)}}{12}\right)^{-24} = 6000$$

$$d^{(12)} \doteq 9.08\%$$

9. An investor puts 100 into Fund X and 100 into Fund Y. Fund Y earns compound interest at the annual rate of $j > 0$, and Fund X earns simple interest at the annual rate of $1.10j$. At the end of 2 years, the amount in Fund Y is equal to the amount in Fund X. Calculate the absolute value of the difference of the amounts in the two funds at the end of 5 years.

(A) 34 (B) 39 (C) 44 (D) 49 (E) 54

After 2 years: $\cancel{100}(1+j)^2 = \cancel{100}(1+(1.1j)(2))$

$\cancel{1}+2j+j^2 = \cancel{1}+2.2j \Rightarrow j^2-0.2j=0$

$\Rightarrow j(j-0.2)=0 \Rightarrow \cancel{j=0}$ or $\underline{j=0.2}$

$\therefore \left|100(1.2)^5 - 100(1+(1.1)(.2)(5))\right| \doteq 39$

10. You are given a loan on which interest is charged over a 4-year period as follows:

I. An effective rate of discount of 6% for the first year
II. A nominal rate of discount of 5% compounded every 2 years for the second year
III. A nominal rate of interest of 5% compounded semiannually for the third year
IV. A force of interest of 5% for the fourth year

Determine the equivalent annual effective rate of interest over the 4-year period.

(A) 0.0500 (B) 0.0525 (C) 0.0550 (D) 0.0575 (E) 0.0600

$$(1+i)^4 = (1-.06)^{-1}\cdot\left(1-\frac{.05}{1/2}\right)^{-1/2}\left(1+\frac{.05}{2}\right)^2\cdot e^{.05}$$

$$\Longrightarrow i \doteq 0.055$$